Is the Asymptotic Speed of Convergence a Good Proxy for the Transitional Growth Path?*

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Abstract

This paper compares transitional dynamics in two alternative R&D non-scale growth models, one with endogenous human capital and the other without. We show that focusing only on the asymptotic speed of convergence to discriminate between the two models' performance can be misleading. Our analysis suggests that a careful study of the entire adjustment paths predicted by alternative growth models starting far away from the balanced growth path is required in order to successfully discriminate among them.

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1 Introduction

The growth literature has devoted considerable time and effort in analyzing the asymptotic speed of convergence predicted by alternative growth models.[1] An important reason for this analysis is its role in establishing stability of the model's long-run equilibrium. Perhaps a more important reason is that, as argue by Ortigueira and Santos (1997) and Eicher and Turnovsky (2001), among many others, a desirable property for growth models is to deliver an asymptotic speed of convergence that is consistent with cross-country empirical studies.[2]

In this paper, we show that the asymptotic speed of convergence in itself maybe a misleading representation of the transitional growth path. For this reason, a careful examination of the entire adjustment path predicted by transitional dynamics is needed to successfully discriminate among alternative growth theories.

More specifically, we study convergence speeds in two versions of the type of *hybrid* non-scale R&D-based growth framework studied by Eicher and Turnovsky (1999a, 1999b, 2001), one without and another with endogenous human capital accumulation.[3] We first compute the asymptotic speeds of convergence predicted by the system of equations that characterize the models' equilibrium dynamics. We find that both the non-scale R&D-based growth model with human capital and the one without human capital predict empirically-supported values.

We discover, however, that this result alone is not very informative about the overall capacity of these two models for reproducing convergence episodes. The reason is that small variations in the asymptotic speed can be related to substantial changes in the initial periods of the adjustment path. More specifically, when we simulate the whole transitional path, we find that, even though both frameworks deliver similar asymptotic speeds of convergence, the dynamics of the model with human capital are able to reproduce important output-convergence experiences such as those of Japan and South Korea much more accurately than the ones of the model without human capital.

We also show that the introduction of human capital makes the asymptotic speed of convergence much less sensitive to changes in some underlying parameter that can be influenced by policy

[1] For example, recent contributions have focused on the effect on the asymptotic convergence speed of income inequality (Zhang (2005)), government financing (Gokan (2003)), or international labor mobility (Rappaport (2005)).

[2] Barro and Sala-i-Martin (1995) report convergence speeds between 0.4% and 6%. In particualr, they vary from 0.4%–3% in Japan, 0.4%–6% in the U.S. and 0.7%–3.4% in Europe. Temple (1998) reports estimates for OECD nations between 1.5% and 3.6%. Authors such as Caselli *et al.* (1996), however, have estimated larger convergence speeds, as high as 10%.

[3] Although other papers such as Keller (1996), Eicher (1996), Funke and Strulik (2000), Lloyd-Ellis and Roberts (2002), and Papageorgiou and Perez-Sebastian (2004, forthcoming) present growth models in which both human capital and technological innovation are endogenous, they are not concerned with the asymptotic speed of convergence.

actions. This finding can offer theoretical support to Barro and Sala-i-Martin's (1995) result that convergence-speed estimates do not vary substantially across different countries or regions.

The remainder of the paper is organized as follows. In section 2, we present the R&D-based model with human capital and study its steady-state predictions. In section 3, we derive the equations used in the transitional dynamics analysis. Sections 4 and 5 obtain numerical results for the asymptotic speed of convergence and examine the entire adjustment path, respectively. Section 6 discusses the findings in more depth, relating them to the existing literature. Section 7 concludes.

2 An R&D-Based Growth Model with Human Capital

The models studied in this paper are extensions of the type of non-scale R&D-based framework studied in Eicher and Turnovsky (1999a, 1999b, 2001). As shown by Jones (1995), this type of framework succeeds in reconciling important properties of the data, such as increasing R&D intensity, with constant output growth rates. We incorporate two modifications: first, we allow for human capital stock to accumulate endogenously over time, and second, technology imitation is costly. As suggested by Bils and Klenow (2000), these modifications make the R&D-based growth model more appropriate for analyzing countries at different levels of economic development.

In this section, we first outline the economic environment under which households and firms operate when human capital accumulation is possible. Then we solve the socially optimal problem. Our exposition is focused on aggregate technologies. The main reason is that the human capital technology incorporated in this paper can not be easily derived from a decentralized setup due to aggregation problems.[4]

2.1 Economic environment

The economy consists of identical infinitely-lived agents, and population grows exogenously at rate n. Agents have preferences only over consumption, and choose to allocate their time endowment in three types of activities: consumption-good production, R&D effort, and human capital attainment.

Our model economy is characterized by the following three equations: First, at period t, output (Y_t) is produced using labor (L_{Yt}) and physical capital (K_t) according to the following aggregate

[4] See footnote 6 for a discussion on the aggregation problem of this approach.

Cobb-Douglas technology:

$$Y_t = A_t^{\xi} \left(h_t \, L_{Yt}\right)^{1-\alpha} K_t^{\alpha} \quad , \quad 0 < \alpha < 1 \, , \;\; \xi > 0, \tag{1}$$

where h_t represents the effectiveness of average human capital level on labor; α is the share of capital; ξ is a technology externality; and A_t is the economy's technical level.

Second, the R&D equation that determines technological progress is given by

$$A_{t+1} - A_t = \mu A_t^{\phi} \left(h_t L_{At}\right)^{\lambda} \left(\frac{A_t^*}{A_t}\right)^{\psi} - \delta_A \, A_t, \quad \phi < 1, \;\; \lambda, \delta_A \in (0,1), \;\; \mu, \psi \geq 0, \;\; A_t^* \geq A_t, \tag{2}$$

where δ_A represents the technology depreciation rate; L_{At} is the portion of labor employed in the R&D sector at time t; A_t^* is the worldwide stock of existing technology at t, which grows exogenously at rate g_{A^*}; ϕ is an externality due to the stock of existing technology; and λ captures the existence of decreasing returns to R&D effort. The above R&D equation is the one proposed by Jones (1995, 2002) plus a *catch-up* term $\left(\frac{A_t^*}{A_t}\right)^{\psi}$, where ψ is a technology-gap parameter. The catch-up term is also consistent with the "relative backwardness" hypothesis of Findlay (1978) that the rate of technological progress in a relatively backward country is an increasing function of the gap between its own level of technology and that of the advanced country.[5]

Third, we have the schooling equation that determines the way by which human capital accumulates. The human capital technology follows Bils and Klenow (2000), who suggest that the Mincerian specification of human capital (Mincer (1974)) is the appropriate way to incorporate years of schooling in the aggregate production function. Following their approach, human capital per capita is given by

$$h_t = e^{f(S_t)} \, , \tag{3}$$

where $f(S_t) = \eta S_t^{\beta}$, $\eta > 0$, $\beta > 0$; and S_t is the labor force average years of schooling at date t. The derivative $f'(S_t)$ represents the return to schooling estimated in a Mincerian wage regression: an additional year of schooling raises a worker's efficiency by $f'(S_t)$.[6]

[5] Nelson and Phelps (1966) are the first to construct a formal model based on the catch-up term. Parente and Prescott (1994) notice that this formulation implies that development rates increase over time (with A_t^*), and provide empirical evidence that is consistent with this implication. Benhabib and Spiegel (1994) find evidence in favor of an R&D equation with imitation in a large sample of countries.

[6] To be fully consistent with the Mincerian interpretation, $H_{jt} = \sum_{i=1}^{L_{jt}} e^{f(s_{it})}$; where s_{it} is the educational attainment of worker i at date t. The mapping between this expression and equation (3) is not straightforward, and has not been addressed by the literature, with the exception of Lloyd-Ellis and Roberts (2002) who perform only balanced-growth path analysis in a finitely-lived agent framework. The difficulty arises because different cohorts can possess different schooling levels. To make both expressions consistent, we could assume that the first generation of agents pins down the workers' educational attainment, and that posterior cohorts are forced to stay in school until they accumulate this educational level. In this way, all workers would have the same years of education (i.e., $s_{it} = S_t$

We assume that, each period, agents allocate time to human capital formation only after output production has taken place.[7] Let L_{Ht} be the total amount of labor invested in schooling in the economy at date t. Assume that at some point in time, say period 1, the average educational attainment equals *zero*. Next period, given that consumers live for ever, the average years of schooling will be $S_2 = \frac{L_{H1}}{L_2}$, where L_t is the labor size at date t. In period 3, $S_3 = \frac{L_{H1}+L_{H2}}{L_3}$, and so on. Hence, the average educational attainment can be written as

$$S_t = \frac{\sum_{j=1}^{t-1} L_{Hj}}{L_t}\,. \tag{4}$$

From equation (4), we can write

$$S_{t+1} = \frac{S_t\, L_t + L_{Ht}}{L_{t+1}}, \tag{5}$$

which in turn implies

$$S_{t+1} - S_t = \left(\frac{1}{1+n}\right)\left(\frac{L_{Ht}}{L_t} - n\, S_t\right). \tag{6}$$

2.2 Social planner's problem

Let C_t be the amount of aggregate consumption at date t. A central planner would choose the sequences $\{C_t, S_t, A_t, K_t, L_{Yt}, L_{At}, L_{Ht}\}_{t=0}^{\infty}$ so as to maximize the lifetime utility of the representative consumer subject to the feasibility constraints of the economy, and the initial values L_0, K_0, S_0, and A_0. The problem is stated as follows:

$$\max_{\{C_t,S_t,A_t,K_t,S_t,L_{Yt},L_{At},L_{Ht}\}} \sum_{t=0}^{\infty} \rho^t \left[\frac{\left(\frac{C_t}{L_t}\right)^{1-\theta} - 1}{1-\theta}\right], \tag{7}$$

subject to,

$$Y_t = A_t^{\xi}\left(e^{f(S_t)} L_{Yt}\right)^{1-\alpha} K_t^{\alpha}\,, \tag{8}$$

$$I_t = K_{t+1} - (1-\delta_k)\, K_t = Y_t - C_t, \tag{9}$$

$$A_{t+1} - A_t = \mu A_t^{\phi}\left(e^{f(S_t)} L_{At}\right)^{\lambda}\left(\frac{A_t^*}{A_t}\right)^{\psi} - \delta_A A_t, \tag{10}$$

for all i) and then $\sum_{i=1}^{L_{jt}} e^{f(s_{it})} = L_{jt}\, e^{f(S_t)}$. However, introducing this into the model would force us to keep track of the different cohorts' years of education across time, thus making the transitional dynamics analysis much more cumbersome, if not impossible. We leave this important issue to future research.

[7]The primary reason for the particular timing of events is mathematical tractability. In particular, this timing allows writing the motion equation of S_{t+1} as a function of S_t and L_{Ht} (see equation (5)). If the timing were the opposite, we would obtain the state variable S_{t+1} as a function of S_t and $L_{H,t+1}$ that could make the optimal control problem significantly more difficult to solve.

$$S_{t+1} - S_t = \left(\frac{1}{1+n}\right)\left(\frac{L_{Ht}}{L_t} - n\,S_t\right), \tag{11}$$

$$L_t = L_{Yt} + L_{At} + L_{Ht}, \tag{12}$$

$$\frac{L_{t+1}}{L_t} = 1+n, \quad \text{for all } t, \tag{13}$$

$$\frac{A^*_{t+1}}{A^*_t} = 1 + g_{A^*}, \tag{14}$$

$$L_0,\ S_0,\ K_0,\ A_0 \ \text{given,}$$

where θ is the inverse of the intertemporal elasticity of substitution; ρ is the discount factor; and δ_k is the depreciation rate of physical capital. Equation (9) is a feasibility constraint as well as the law of motion of the stock of physical capital; it states that, at the aggregate level, domestic output must equal consumption plus physical capital investment, I_t. Equation (12) is the labor constraint; the labor force – that is, the number of people employed in the output and the R&D sectors – plus the number of people in school must be equal to population.

Solving this dynamic optimization problem obtains the Euler equations that characterize the optimal allocation of labor in human capital investment, in R&D investment, and in consumption/physical capital investment respectively as follows:

$$\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}} = \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[1 + f\,'(S_{t+1})\left(\frac{L_{Y,t+1}}{L_{t+1}} + \frac{L_{A,t+1}}{L_{t+1}}\right)\right], \tag{15}$$

$$\begin{aligned}\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}} &= \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{\lambda\left[A_{t+1}-(1-\delta_A)A_t\right]}{L_{At}} * \\ &* \left\{\frac{\xi Y_{t+1}}{A_{t+1}} + \left[1-\delta_A+(\phi-\psi)\left(\frac{A_{t+2}-(1-\delta_A)A_{t+1}}{A_{t+1}}\right)\right]\left[\frac{\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}}{\frac{\lambda(A_{t+2}-(1-\delta_A)A_{t+1})}{L_{A,t+1}}}\right]\right\},\end{aligned} \tag{16}$$

$$\left(\frac{C_t}{L_t}\right)^{-\theta} = \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\left[\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta_k)\right]. \tag{17}$$

At the optimum, the planner must be indifferent between investing one additional unit of labor in schooling, R&D, and final output production. The LHS of equations (15) and (16) represent the return from allocating one additional unit of labor to output production. The RHS of equation (15) is the discounted marginal return to schooling, taking into account labor growth. The RHS term in brackets arises because human capital determines the effectiveness of labor employed in output production as well as in R&D. The RHS of equation (16) is the return to R&D investment. An additional unit of R&D labor generates $\frac{\lambda[A_{t+1}-(1-\delta_A)]A_t}{L_{At}}$ new ideas for new types of producer durables. Every new design increases next period's output by $\frac{\xi Y_{t+1}}{A_{t+1}}$ and R&D production by $\frac{dA_{t+2}}{dA_{t+1}}$ *times* $\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[\frac{\lambda(A_{t+2}-(1-\delta_A)A_{t+1})}{L_{A,t+1}}\right]^{-1}$; where the term $\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[\frac{\lambda(A_{t+2}-(1-\delta_A)A_{t+1})}{L_{A,t+1}}\right]^{-1}$ gives the value of one additional design that equalizes labor wages across sectors. Euler equation (17) is standard and states that the planner is indifferent between consuming one additional unit of output today and converting it into capital (thus consuming the proceeds tomorrow).

2.3 Steady-state growth

We now derive the model's balanced-growth path. Solving for the interior solution, equation (12) implies that in order for the labor allocations to grow at constant rates, L_{Ht}, L_{Yt} and L_{At} must all increase at the same rate as L_t. This means that the ratio $\frac{L_{Ht}}{L_t}$ is invariant along the balanced-growth path. Hence, equation (11) implies that, at steady-state (ss), S_{ss} is constant and equals

$$S_{ss} = \frac{u_{H,ss}}{n}, \tag{18}$$

where $u_{H,ss} = \left.\frac{L_H}{L}\right|_{ss}$. Equation (18) shows that along the balanced-growth path, the economy invests in human capital just to provide new generations with the steady-state level of schooling.

Let lower case letters denote per capita variables, and $g_x = G_x - 1$ denote the growth rate of x. The aggregate production function, given by equation (8), combined with the steady-state condition $g_{Y,ss} = g_{K,ss}$ delivers the gross growth rate of output as a function of the gross growth rate of technology as

$$G_{Y,ss} = (G_{A,ss})^{\frac{\xi}{1-\alpha}}(1+n). \tag{19}$$

Since $G_{A,ss}$ is a constant, it follows from equation (2) that

$$G_{A,ss} = \left[(1+n)^{\lambda}(G_{A^*,ss})^{\psi}\right]^{\frac{1}{1+\psi-\phi}}. \tag{20}$$

Equation (20) shows the relationship between the technology frontier growth rate and the technology growth rate of the model economy. Since $\frac{\psi}{1+\psi-\phi} < 1$, it is easy to show that there is a unique point at which

$$G_{A,ss} = G_{A^*,ss} = (1+n)^{\frac{\lambda}{1-\phi}}. \tag{21}$$

We focus on a special case: we suppose that $G_{A^*,ss}$ is given by expression (21) and, therefore, so is $G_{A,ss}$.[8] This in turn implies that

$$G_{Y,ss} = G_{C,ss} = G_{K,ss} = (1+n)^{\frac{\lambda\xi}{(1-\alpha)(1-\phi)}}. \tag{22}$$

[8] We could assume that a technology leader shifts outward the world technological frontier according to equation (2) which now reduces to

$$A^*_{t+1} - A^*_t = \mu A^{*\phi}_t (h^*_{At}L^*_{At})^{\lambda} - \delta_A A^*_t,$$

Consistent with Jones (1995, 2002) our balanced-growth path is free of *scale effects.* The reason why the model's long-run growth is equivalent to that of Jones even in the presence of a schooling sector, is that at steady state the mean years of education, S_t, reaches a constant level S_{ss}.[9]

3 Transitional Dynamics

We now turn attention to the transitional-dynamics predictions of the model presented above (thereafter, *model with H*). Our main goal is to compare these predictions to the ones obtained from an identical model but without human capital. This second type of framework (thereafter, *model w/o H*) can be obtained by simply removing human capital from the above setup, and corresponds to the class of two-sector non-scale growth model studied by Jones (1995), Eicher and Turnovsky (1999a) and Perez-Sebastian (2000), among others.

In order to generate the system of equations that can help us study transitional dynamics, we need to redefine variables so that their values remain constant at steady state. In particular, long-run growth rates given by equations (21) and (22) suggest that we normalize variables C_t, K_t, and Y_t by the term $L_t^{\sigma_k}$, and the variable A_t by $L_t^{\sigma_A}$, where $\sigma_k = \lambda\xi/[(1-\phi)(1-\alpha)] + 1$, and $\sigma_A = \lambda/(1-\phi)$. We can then rewrite consumption, physical capital, output and technology as $\hat{c}_t = C_t/L_t^{\sigma_k}$, $\hat{k}_t = K_t/L_t^{\sigma_k}$, $\hat{y}_t = Y_t/L_t^{\sigma_k}$and $\hat{a} = A_t/L_t^{\sigma_A}$, respectively.

Next, we present the normalized system for the *model with H* and for the *model w/o H*. We also derive the equation that obtains the asymptotic speed of convergence.

3.1 The normalized systems for the model with H

Using equation (15) gives

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta}\left(\frac{u_{Y,t+1}}{u_{Yt}}\right)(1+n)^{(\theta-1)(\sigma_k-1)+1}\left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right) = \rho\left[f'(S_{t+1})\,(u_{Y,t+1}+u_{A,t+1})+1\right]. \quad (23)$$

where now $\frac{A_t^*}{A_t} = 1$ as imitation is not possible at the frontier; and $*$ denotes the value which variables take in the leading country. In such case $G_A^* = 1 + g_A^* = (1+n^*)^{\frac{\lambda}{1-\phi}}$ as in Jones (1995, 2002). Assuming that $n = n^*$, and substituting G_A^* into equation (20) delivers equation (21).

[9] It is not, however, obvious how to think about years of schooling within infinite horizon frameworks. In particular, it is not clear that a technology that delivers a constant steady-state level of schooling should be preferred to another one that generates an increasing value for this variable. See Papageorgiou and Perez-Sebastian (2005) for a deeper discussion on this issue, and the advantages of our human capital technology within the infinitely-lived-agent context.

From equation (16) we obtain

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta}\left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right)\left(\frac{u_{Y,t+1}}{u_{Yt}}\right) = \frac{\rho\left(g_{At}+\delta_A\right)}{G_{At}\left(1+n\right)^{(\theta-1)(\sigma_k-1)}}\left(\frac{u_{A,t+1}}{u_{At}}\right) * \\ * \left[\left(\frac{\lambda\xi}{1-\alpha}\right)\left(\frac{u_{Y,t+1}}{u_{A,t+1}}\right)+\left(\frac{1-\delta_A}{g_{A,t+1}+\delta_A}\right)+(\phi-\psi)\right]. \quad (24)$$

From the R&D equation (2), we derive G_{At} as

$$G_{At} = \frac{A_{t+1}}{A_t} = 1-\delta_A+\mu\hat{a}_t^{\phi-1}\left[e^{f(S_t)}u_{At}\right]^{\lambda}T_t^{\psi}, \quad (25)$$

where $T=\frac{A_t^*}{A_t}$. Finally, from equation (17) we obtain

$$\frac{1+n}{\rho}\left[\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)(1+n)^{\sigma_k-1}\right]^{\theta} = \alpha\frac{\hat{y}_{t+1}}{\hat{k}_{t+1}}+(1-\delta_K). \quad (26)$$

The system that determines the dynamic equilibrium normalized allocations is given by the conditions associated with three control and four state variables as follows:

Control Variables:

1. Euler equation for population share in schooling, u_{ht}: Eq. (23).
2. Euler equation for population share in R&D, u_{At}: Eq. (24).
3. Euler equation for normalized consumption, $\hat{c}_t$: Eq. (26).

Subject to the population constraint $u_{Yt}=1-u_{At}-u_{ht}$.

State Variables:

1. Law of motion of human capital, S_t: Eq. (6).
2. Law of motion of the technology gap, T_t:

$$T_{t+1} = T_t\left(\frac{G_{A^*t}}{G_{At}}\right). \quad (27)$$

3. Law of motion of normalized physical capital, $\hat{k}_t$:

$$\hat{k}_{t+1}(1+n)^{\sigma_k} = (1-\delta_K)\hat{k}_t+\hat{y}_t-\hat{c}_t, \quad (28)$$

4. Law of motion of normalized technology stock, $\hat{a}_t$:

$$\hat{a}_{t+1}(1+n)^{\sigma_A} = \hat{a}_t G_{At}, \quad (29)$$

where G_{At} is given by expression (25), $G_{A^*t}=G_{A,ss}$ for all t, and

$$\hat{y}_t = \hat{a}_t^{\xi}\hat{k}_t^{\alpha}\left[e^{f(S_t)}\,u_{Yt}\right]^{1-\alpha}. \quad (30)$$

3.2 The normalized system for the model w/o H

The model economy is now characterized by two control variables (consumption and R&D-labor) and three state variables (physical capital, technology stock, and technology gap). It is straightforward to show that the system of equations that determines the dynamics in the economy without schooling sector consists of Euler conditions (24) and (26), and motion equations (27), (28) and (29), subject to $f(S) = 0$, the population constraint $u_{Yt} = 1 - u_{At}$, $G_{A^*t} = G_{A,ss}$, and equations (25) and (30).

3.3 Asymptotic speed of convergence

To compute the asymptotic speed of convergence, we need to linearize the normalized system of Euler and motion equations around the steady state, and express the resulting system as follows:

$$\vec{x}_{t+1} = D\,\vec{x}_t,$$

where $\vec{x}$ is the vector consisting of the state and control variables; and D is the matrix of first derivatives $(\partial x_{i,t+1}/\partial x_{jt})$ $\forall i, j$ evaluated at the steady state, with x_i being the i^{th} component of vector $\vec{x}$. In the *model with H*, the transpose of this vector is $\vec{x}'_t = (\hat{c}_t, u_{At}, u_{Ht}, \hat{a}_t, \hat{k}_t, T_t, S_t)$, whereas for the *model w/o H*, $\vec{x}'_t = (\hat{c}_t, u_{At}, \hat{a}_t, \hat{k}_t, T_t)$.

Second, we compute the eigenvalues associated with the matrix D. Convergence speed is obtained by the largest eigenvalue (denoted as *eigen*) among those contained in the unit circle. Adapting the measure proposed by Eicher and Turnovsky (1999b), asymptotic speed of convergence (denoted as *asc* hereon) of normalized variable $\hat{y}$ is given by

$$asc(\hat{y}) = -\frac{(\hat{y}_{t+1} - \hat{y}_t) - (\hat{y}_{t+1,ss} - \hat{y}_{t,ss})}{\hat{y}_t - \hat{y}_{t,ss}} = 1 - eigen.$$

Given that we are primarily interested in output *per worker*, $\frac{Y}{L_A+L_Y} = \hat{y}A^{\frac{\xi}{1-\alpha}}(u_A + u_Y)^{-1}$ (call it y^w), it is easy to show that, when labor shares converge faster than output, its *asc* can be obtained as[10]

$$asc\,(y^w) = (1 - eigen)G_{y,ss} - g_{y,ss}. \tag{31}$$

[10] This expression actually gives the *asc* of output per capita, $asc(y)$, where $y = Y/L$. In the *model w/o H*, both output per worker and output per capita coincide. But this is not the case in the *model with H*. However, $asc(y)$ serves as an excellent approximation for $asc(y^w)$ in our parameterization, even with S accumulation. The reason is that, as will become clear later, the speed of convergence of the labor force is larger than the one of output and, as a consequence, the latter speed is not asymptotically affected by the former.

Table 1: Benchmark parameter values for the *model w/o H*

α	0.36	ξ	0.1	ρ	0.96	ψ	0.16
δ_K	0.06	λ	0.5	θ	1	T_{ss}	1
δ_A	0.01	ϕ	0.927	n	0.015		

4 Numerical Results for the Asymptotic Speed of Convergence

To highlight the changes brought by the introduction of human capital into the model, we first present results from the *model w/o H* followed by related results from the *model with H*. Since, closed-form solutions do not exist neither for the general analysis of the model's transitional dynamics nor for matrix D, we resort to numerical methods.

4.1 Parameterization

Table 1 describes our benchmark economy for the *model w/o H*. For the sake of comparability, the parameter values are those chosen by Eicher and Turnovsky (1999b, 2001). The only exceptions are the parameters ϕ and ψ related to the R&D technology. In particular, these authors consider an economy without a schooling sector and without imitation, assigning a value of 0.5 to ϕ, and of *zero* to ψ. They show that in this environment the stable manifold is two dimensional. Hence, the adjustment path is asymptotically stable and unique. Furthermore, growth rates and convergence speeds can, as a consequence, vary across time and variables. For this parameterization but with $\phi = 0.5$, and $\psi = 0$, our numerical methods obtain an $asc(y^w)$ of 0.0179.[11] This is, actually, a major finding of Eicher and Turnovsky (1999b, 2001). That is, going from the neoclassical one-sector growth model to a two-sector non-scale growth model substantially reduces the asymptotic speed of convergence from about 7 percent to about 2 percent.[12]

As reported in Table 1, we instead choose $\phi = 0.927$ and $\psi = 0.16$. That is, we consider a *model w/o H* with an R&D sector that exhibits increasing returns in knowledge and labor, and imitation. The reason to assign a larger value to ϕ is that we want to generate reasonable values for the steady-state per capita output growth rate. Taking $g_{y,ss} = 1.6$ percent, the average g_y in Bils and Klenow's (2000) 91-country sample, implies that $\phi = 0.927$ through equation (22), for given

[11] All numerical results were obtained using *MATHEMATICA*. Programs are available by the authors upon request.

[12] These authors employ a continuous-time version of the model that provides slightly larger speeds than our discrete-time approach. In particular, for the benchmark economy, the continuous-time analog would imply $asc(y^w) = 0.0184$. The slightly larger speed implied by continuous-time holds across all the models considered in our paper.

Table 2: Parameter values for the *model with H*

α	0.36	ξ	0.1	ρ	0.96	ψ	0.16	T_{ss}	1
δ_K	0.06	λ	0.5	θ	1	η	0.69	S_{ss}	12.03
δ_A	0.01	ϕ	0.927	n	0.015	β	0.43	$g_{y,ss}$	0.016

values of λ, n, ξ, and α.

A value of ψ greater than *zero*, in turn, allows reconciling a reasonable $g_{y,ss}$ with fast development experiences, as demonstrated by Perez-Sebastian (2000). Otherwise, the two-sector *hybrid* non-scale R&D-based growth model delivers implausibly low converge speeds, with half lives in the hundred of years.[13] A value of 0.16 for ψ is within the calibrated values that we obtain later on.

Next, we choose parameter values for the *model with H* that includes schooling and imitation. To do this, we need to calibrate the human capital technology. Following Bils and Klenow (2000), we assume that

$$f(S) = \eta S^{\beta}, \ \ \eta > 0, \ \beta > 0. \tag{32}$$

Then using Psacharopoulos' (1994) cross-country sample on average educational attainment and Mincerian coefficients we estimate η and β. Given equation (32), we can construct the log-linear regression equation

$$\ln(Mincer_i) = a + b \ln S_i + \varepsilon_i, \tag{33}$$

where $Mincer_i = f'(S_i)$ is the estimated Mincerian coefficient for country i; a and b equal $ln(\eta\beta)$ and $(\beta - 1)$, respectively; and ε_i is a random disturbance term. We obtain estimates of $\eta = 0.69$ and $\beta = 0.43$, both significantly different from zero at the 1 percent level, that are very similar to those obtained by Bils and Klenow (2000).

Table 2 presents the benchmark economy for the *model with H*. It is composed of the benchmark parameter values for the *model w/o H* (Table 1) plus the human capital technology parameters ($\eta = 0.69$, $\beta = 0.43$). Given the above values, equations (15), (18) and (22) imply that the steady-state average educational attainment is 12.03 years, close to the 2000 U.S. figure of 12.05 obtained by Barro and Lee (2001). For this economy, the stable manifold is pinned down by three eigenvalues that are contained within the unit circle. That is, the transition is characterized by a three-dimensional stable saddle-path which in turn implies that the adjustment path is asymptotically

[13] This result was originally shown by Jones (1995). For example, with the benchmark parameterization but taking $\phi = 0.5$ and $\psi = 0$, the *model w/o H* generates $g_{y,ss} = 0.0023$. Taking $\phi = 0.9$, $g_{y,ss}$ rises to 0.012, but the implied *asc* falls to 0.0007, clearly an implausibly low value.

Table 3: Asymptotic speed of convergence for different parameterizations

	Benchmark	$\delta_A = 0.1$	$\psi = 0.25$	$\lambda = 0.75$
Model w/o H	0.0196	0.0410 (2.1)	0.0326 (1.7)	0.0320 (1.6)
Model with H	0.0132	0.0172 (1.3)	0.0168 (1.3)	0.0075 (0.6)

Note: Numbers inside parenthesis give the ratio with respect to benchmark value in same row.

stable and unique.[14]

4.2 Calibration results for the asymptotic speed of convergence

Table 3 presents results. In the *model w/o H*, $asc(y^w)$ is equal to 0.0196 for the benchmark parameterization (Table 1), a value consistent with empirical estimates found in the literature. Let us now take a first look at how changes in parameters that may be policy-dependent affect *asc* (further robustness analyses exist in section 4.3). The depreciation rate of technology, δ_A, can arguably be affected by patent laws. In particular, if there is a reduction in the legal life of patents, obsolescence/depreciation would most likely increase. The technology-gap parameter, ψ, could also be considered a policy parameter because it is likely related to barriers to technology adoption (see, e.g. arguments in Parente and Prescott (1994)). A successful policy to enhance technological adoption could cause an increase in ψ. Finally, diminishing returns to R&D investment are in part a consequence of races between R&D firms. Regulation conducted to induce cross-firm R&D collaborations could weaken these diminishing returns, thus increasing λ.

The first row of Table 3 also reports the *asc* for the *model w/o H* under different values of δ_A, ψ and λ. We consider that δ_A increases from 0.01 to 0.1, another empirical-supported value (see Caballero and Jaffe (1993)). The *model w/o H* predicts in this case a relatively large increase from 0.0196 to 0.0410 (i.e., a 2.1 fold). When ψ increases from 0.16 to 0.25, the consequence is that $asc(y^w)$ becomes 0.033. Finally, if λ increases from 0.5 to the 0.75 estimated by Jones and Williams (2000) then $asc(y^w)$ rises to 0.032.

The second row of Table 3, in turn, reports results for the *model with H*. In this framework, *asc* is equal to 0.0132 for the benchmark values (Table 2). Note that even though this convergence speed is lower than the 0.0196 provided by the *model w/o H*, it is still well within empirical estimates. The reduction in the convergence speed occurs because of the additional schooling sector present in our model. A new sector implies that the same amount of available labor must now be allocated

[14]This result is robust to reasonable changes in the parameter values.

among three (rather than two) sectors, which makes state variables move more slowly towards the balanced-growth path.

The results for the *model with H* with alternative values of δ_A, ψ and λ, offer an insight worth noting. The *asc* becomes less responsive to changes in our policy parameters when we introduce human capital in the R&D-based growth model. When δ_A increases from 0.01 to 0.1, the *model with H* predicts a small increase in $asc(y^w)$ from 0.0132 to 0.0172 (i.e., a 1.3 fold), much smaller than the change produced in the *model w/o H*. A lower sensitivity of the *asc* is also obtained if the technology-gap parameter ψ varies from 0.16 to 0.25. In particular, $asc(y^w)$ now increases from 0.0132 to 0.0168. Finally, the same is true if λ rises from 0.5 to 0.75. In this last case, however, the effect goes in the opposite direction. Now, the *asc* declines with λ from 0.0132 to 0.0075, that is, a fall of 0.0057, against an increase of 0.0144 in the *model w/o H*. The reason for the weaker response of the asymptotic speed in the *model with H* is again the one given above. The presence of the additional sector implies a lower allocation of resources to each of the different activities, thus reducing the impact of external shocks.

This low sensitivity of the convergence speed to changes in the parameter values is consistent with Barro and Sala-i-Martin's (1995) finding that estimated convergence speeds do not vary much across different countries or regions. However, our result does not necessarily imply that policy actions have a small impact on the transition process, as Barro and Sala-i-Martin's result has been interpreted. Far away from the balanced-growth path, policy may have a larger effect on the speed of convergence over subsequent periods because the model allows the convergence speed to vary across time. It is also important to notice that Barro and Sala-i-Martin's finding is obtained for a fairly homogenous group of wealthy regions – namely, U.S. states, European regions, and Japanese prefectures – which are probably close to their steady states.

4.3 Robustness

In this section, we extend the sensitivity analysis of our results (presented in Table 3) to alternative changes in key parameter values. It is known from Jones (1995) and Eicher and Turnovsky (1999b, 2001) that the type of non-scale R&D growth model that we use is especially sensitive to the returns to scale and the shares of technology and labor in the R&D sector. For this reason, we focus on the R&D technology parameters ϕ and λ. We study how changes in their values affect the predicted *asc*, and the impact of policy actions.

The empirical literature does not offer much guidance in choosing a reasonable value for tech-

Table 4: Robustness analysis of the predicted asymptotic speed of convergence

	Cases	Asymptotic speed of convergence†			
		Benchmark	$\delta_A = 0.1$	$\psi = 0.25$	$\lambda = 0.75$
Model w/o H	$\phi = 0.75$	0.0208	0.0643 (3.1)	0.0260 (1.3)	0.0378 (1.8)
	$\phi = 0.50$	0.0240	0.0902 (3.8)	0.0273 (1.1)	0.0434 (1.8)
	$\lambda = 0.75$	0.0320	0.0496 (1.6)	0.0514 (1.6)	–
	$\lambda = 0.25$	0.0090	0.0313 (3.5)	0.0154 (1.7)	–
Model with H	$\phi = 0.75$	0.0194	0.0291 (1.5)	0.0232 (1.2)	0.0229 (1.2)
	$\phi = 0.50$	0.0228	0.0320 (1.4)	0.0254 (1.1)	0.0276 (1.2)
	$\lambda = 0.75$	0.0075	0.0092 (1.2)	0.0095 (1.3)	–
	$\lambda = 0.25$	0.0089	0.0246 (2.8)	0.0150 (1.7)	–

† Asymptotic speeds obtained for benchmark parameter values, except for the parameters shown in corresponding row and column. Values in parentheses present the ratio with respect to the benchmark value in the same row.

Table 5: Output, Capital and Schooling in Japan and S. Korea

Country		1960	1963	1990
Japan	Y per worker (%)**	20.6		60.3
	K per worker (%)**	16.9		104.6
	S (years)	10.2		11.0*
S. Korea	Y per worker (%)**		11.0	42.2
	K per worker (%)**		11.6	50.2
	S (years)		3.2	7.7*

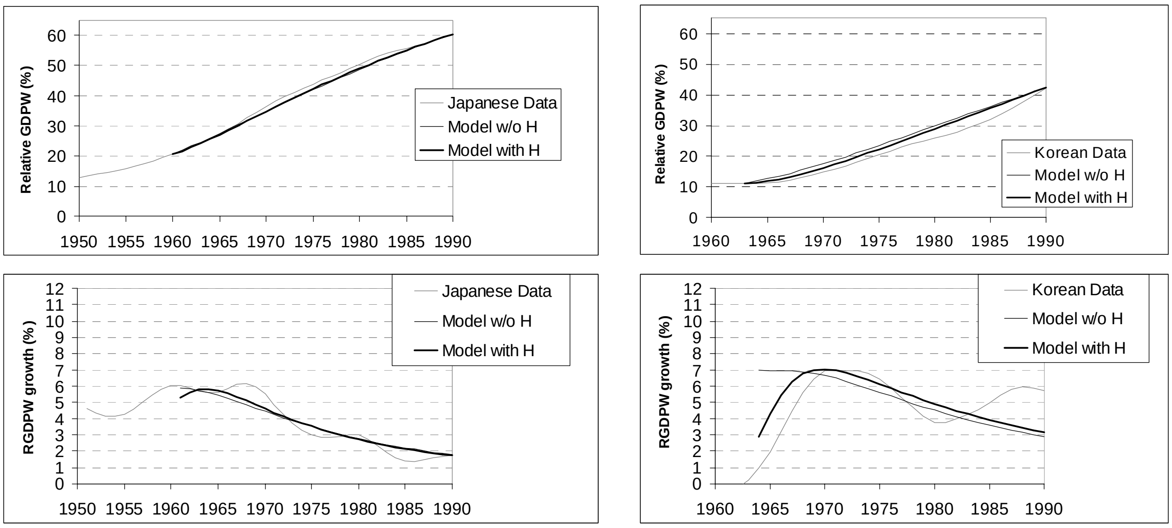
* 1987 figures.

** Levels relative to their U.S. counterparts.

nology externality ϕ. In our benchmark case, the value $\phi = 0.927$ is pinned down by the balanced-growth equation (22). Some authors like Eicher and Turnovsky (1999b, 2001), however, argue that $\phi = 0.927$ may be too large. The first and second rows of results in Table 4 show predicted asymptotic speeds for the *model w/o H* when ϕ equals 0.75 and 0.5, respectively. The fifth and sixth rows, in turn, show respective values for the *model with H*. Focusing on these results we readily see that the qualitative outcome is the same as before. Both models generate empirically-supported benchmark convergence speeds with the predicted values for the *model with H* being always lower and less sensitive to changes in policy parameters than the ones for the *model w/o H*.

Estimates of the labor share in the R&D sector, λ, found in the literature vary from 0.2 (Kortum (1993)) to 0.75 (Jones and Williams (2000)). We examine how convergence speeds change when we replace our baseline value of $\lambda = 0.5$ with the more extreme values $\lambda = 0.25, 0.75$. As shown in

Figure 1: Adjustment paths for Japan and S. Korea



rows three, four, seven and eight in Table 4, once again we obtain qualitatively similar results with the sensitivity of predictions still remaining lower in the *model with H*.

5 Examination of the Entire Adjustment Path

In the previous section, we have shown that the two models that we compared predicted asymptotic speeds that comply with the empirical evidence. In this sense, we could conclude that the above results suggest that the *model with H* does not represent an improvement over the *model w/o H* to reproduce convergence experiences. Furthermore, if we take into account that some estimates of the speed of convergence, such as in Caselli *et al.* (1996), obtain relatively large convergence-speed estimates, as high as 10 percent, the lower speed predicted by the *model with H* could be even interpreted as a drawback of the model.

5.1 Reproducing the South Korean and Japanese growth experiences

Next, we demonstrate that making conclusions about the transitional dynamics performance of a growth model using as a measure only the asymptotic speed of convergence can be quite misleading. We do this by comparing the capacity of the two models presented in the previous sections to reproduce country-specific changes in some key variables in two important growth experiences; namely those of South Korea and Japan.[15] The reason for considering the modern growth performance of South Korea and Japan is because although these two countries have experienced unprecedented output growth, they also represent two distinctly different development experiences. This is clearly shown in Table 5 where the main engine of growth in Japan seems to have been physical capital accumulation complemented by a very important technological catch-up process, whereas human capital accumulation seemed to have played the dominant role in the development process of South Korea.

Taking the model to the data requires assigning a value to ψ. Here, we follow Parente and Prescott (1994), and assume that countries may differ in their degrees of technology adoption barriers. For simplicity, we suppose that these barriers affect the value of the parameter ψ. To obtain its economy-specific value, we calibrate the parameter ψ to each country's output data. Because we focus on two nations, Japan and South Korea, the value that the parameter ψ takes will be the one that makes transitional dynamics able to reproduce the output per worker evolution between 1960 and 1990 in Japan, and between 1963 and 1990 in South Korea.[16]

The initial values of the stock variables and output data used to calibrate ψ are presented in Table 5.[17] The *model with H* requires $\psi = 0.131$ to induce Japan's average growth between 1960 and 1990, and $\psi = 0.162$ to produce the South Korean output numbers. The *model w/o H* requires $\psi = 0.10$ for the Japanese development experience, and $\psi = 0.074$ for the South Korean development experience.

5.2 Adjustment paths for output levels and growth rates

The adjustment paths predicted by both models for the level and growth rates of relative GDP per worker (RGDPW) are depicted in Figure 1. The two top panels imply that, although the *model with H* does slightly better, both frameworks generate output paths that replicate fairly well the Japanese and the South Korean data. This was expected knowing that both models generate a similar *asc* for output. However, the message from the bottom two panels is different. The *model with H* does a much better job because it predicts that output per worker growth rates do not pick at the beginning of the adjustment path but later on. This is an important feature that

[15] Once again, we resort to numerical approximation techniques to simulate transitional dynamics. The method used and measures of its accuracy are provided in the Appendix.

[16] Japan's rapid convergence toward U.S. income levels actually started right after WWII. Unfortunately, the Japanese Education Department does not possess estimates of the average educational attainment before 1960.

[17] All relative measures in the paper are with respect to U.S. levels. Additionally, we follow Parente and Prescott (1994) and smooth all data series using the Hodrick-Prescott filter with the smoothing parameter equal to 25.

characterizes the output-convergence phenomenon as Easterly and Levine (1997), among others, show. Because of this, it can be argued that the *model with H* represents a better theory to explain convergence experiences than the *model w/o H*.

5.3 Adjustment paths for other variables

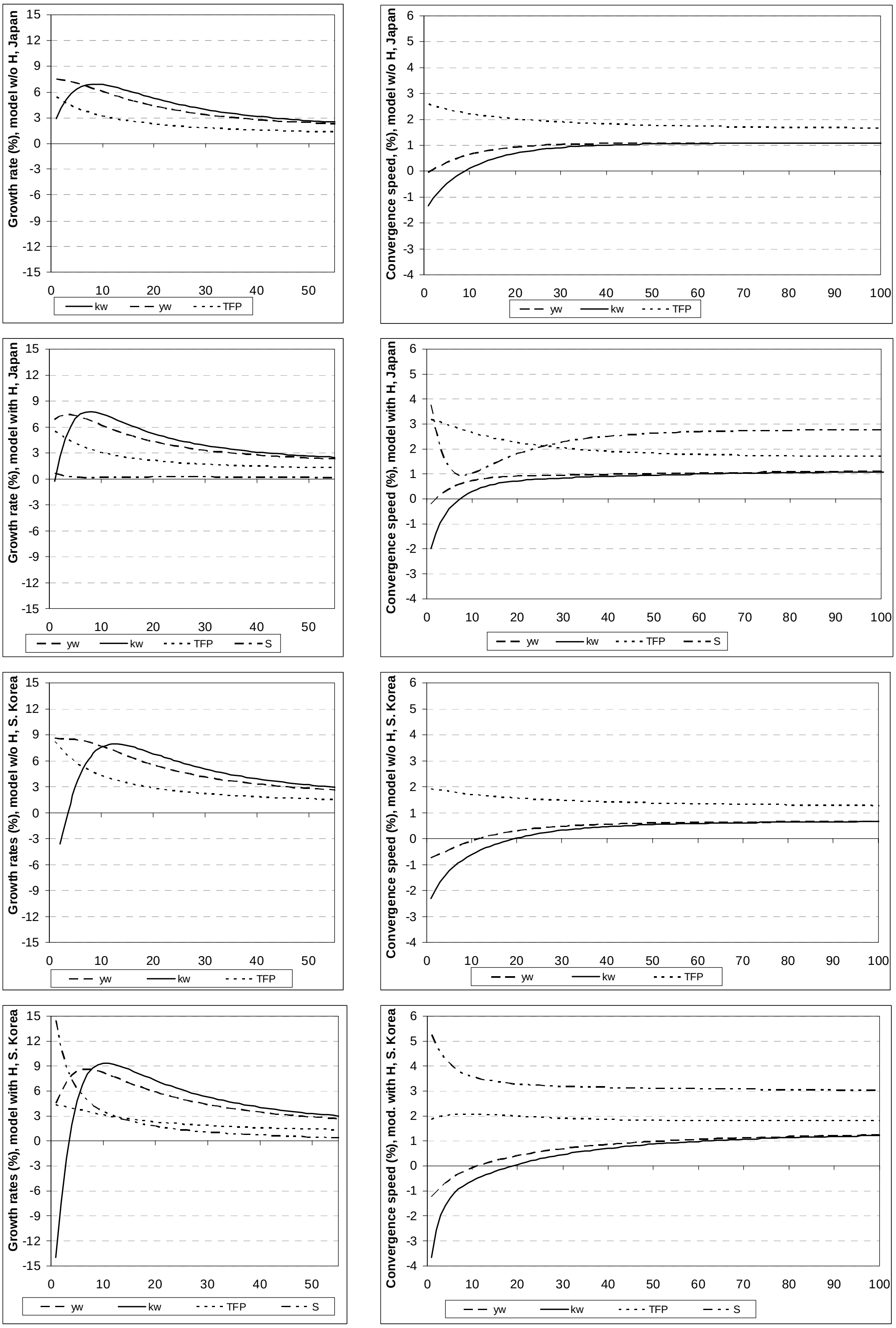
Next we study the time path of growth rates and convergence speeds for some key variables such as output per worker (y^w), capital per worker (k^w), total factor productivity (TFP), and the average educational attainment (S). These results are presented in Figure 2. The first and second rows depict the Japanese paths for the *model w/o H* and the *model with H*, respectively, whereas the third and fourth rows depict the South Korean paths for the *model w/o H* and the *model with H*, respectively.

We observe that different variables show clearly different growth-rate and convergence-speed paths. Focusing first on the growth rate paths (first column of charts), there are a few points worth noting. Growth rates of y^w are initially smaller in the *model with H*. The reason is that schooling is the only activity that enhances the productivity of the other two sectors, and consequently it is optimal for the economy to invest heavily in human capital at the beginning of the adjustment path, borrowing resources mainly from the consumption-good sector. From the same reasoning, physical capital suffers a slightly larger initial fall in the model with schooling, and accumulates at a faster rate during the first few periods following the evolution of output. The large initial differences between the growth rates of output and capital in both models are due to consumption smoothing that exerts a downward pressure on the investment share as output declines therefore causing physical capital to grow at a much lower rate than output during the first few periods.

TFP growth in the *model with H* is faster along the whole adjustment path. This occurs because of the larger value of the catch-up parameter ψ that raises R&D productivity. The difference is larger for South Korea because the value of the parameter ψ required is also larger.

Most importantly, the main difference between the two models is due to the effect of population movements across sectors. Specifically, output increases with the amount of labor devoted to final-good production, and at the same time additional labor deflates output per worker. As a consequence, output decreases with the number of students that leave school and enter the labor force, and increases as R&D effort declines because part of the R&D labor is reallocated to the final output sector. Along the *model with H* transitional dynamics, the deflating effect of students entering the labor force is larger at the beginning, and rapidly decreases as the economy approaches

Figure 2: Growth and convergence rates for key variables



the steady state, which generates a fast declining pattern of labor force growth. This effect is what mainly induces the initially rising output growth rates.

The second column of charts in Figure 2 presents the different convergence speeds. The education sector is the one that converges more quickly, primarily because it is the only activity that complements all other sectors. TFP shows the second fastest convergence speed, whereas the slowest input is physical capital. We also see that physical capital accumulation is the main determinant of $asc(y^w)$. In addition, because the convergence speeds of TFP and S are always faster than those of y^w and k^w, they have a negligible impact on the output per worker convergence speed when this is sufficiently close to the steady state.

The above exercise reveals an important reason for why there may be substantial differences between the asc and the transitional path predictions, when used to discriminate between alternative growth theories. This is that the asc is not affected by important variables that shape the whole adjustment path. For example, we have shown that the success of the *model with H* in explaining the y^w growth rate path is due, in part, to the contribution of labor force movements. Labor force changes have no influence, however, on determining the $asc(y^w)$. Recall that education is the main driving force of cross-sector labor reallocations. This implies that its convergence speed is very similar to the one of the labor force. In particular, for the South Korean and Japanese cases, the *model with H* gives $[asc(S), asc(u_A + u_Y)]$ equal to $[2.9, 3.0]$ and $[2.7, 2.8]$, respectively. As a consequence, labor force changes have an insignificant impact on $asc(y^w)$, even though they are a main determinant of the transitional path.

6 Discussion

The main finding from this work is that focusing on the asymptotic speed of convergence as a key measure to assess a model's capacity to reproduce a country's transitional growth performance may be quite misleading. Put differently, focusing on the asymptotic speed of convergence can give only an incomplete picture of the transitional adjustment path. In this section, we address this issue in greater depth and relate our findings to the existing literature.[18]

This paper shows that unless we consider the whole transitional adjustment path we can not make positive statements about past performance or commit to sound policy recommendations. We obtain this result by means of a calibration exercise of a modified R&D growth model (to allow for inclusion of human capital) that compares the remarkable, albeit different, growth experiences

[18] We thank a referee whose comments were invaluable for this section.

of Japan and South Korea. It is important to note that there exists related work in the literature that supports our finding using primarily numerical exercises. A prominent example is the work of Eicher and Turnovsky (1999b, 2001), and related work by Turnovsky (2004) and Alvarez-Cuadrado *et al.* (2004). Their results also support that the asymptotic convergence speed can be a poor representation of the transitional path.

More specifically, Eicher and Turnovsky (1999b) show that in the case where the transitional dynamics are described by a first order linear system, this yields a constant speed of convergence equal to the stable eigenvalue. On the other hand, in their two-sector non-scale growth model, dynamics are characterized by a second order system and, as a consequence, the convergence speed is a weighted average of two eigenvalues. Over time, the weight shifts more toward one of the eigenvalues, converging to it, asymptotically. The same applies in our paper where, in one case, the stable manifold is three dimensional.

As Eicher and Turnovsky show in subsequent work, the consequence is that the speed of convergence can follow various patterns, and even show dramatic changes during the course of the transition. Eicher and Turnovsky (2001, pp.102-106) plot several different time paths of the speed of convergence. Interestingly, in some cases they find that the rate of convergence is slow, so that the asymptotic rate may be fairly irrelevant for understanding short-run dynamics. They also show that, other times, the asymptotic speed of convergence can even become infinite (2001, Figure 4d, p.104). In addition, Alvarez-Cuadrado *et al.* (2004) show that complex eigenvalues can also rise and pose further problems with the speed of convergence. In particular, these authors show that under their preference specifications, the two stable roots are complex, indicating that the stable adjustment path exhibits cyclical behavior. However, since the imaginary component is small, the periodicity of the cycles are extremely long so that practically the transitional paths can be thought of as non-cyclical. With the transitional path cycling around its steady state, some care is needed in adapting the measure. In sum, previous work had somewhat suggested as well that looking only at the asymptotic speed of convergence is clearly missing much of the picture.

Another problem with using only the asymptotic value is that the benchmark of 2 percent originally estimated by the empirical growth literature has been challenged several times, obtaining a range of plausible estimates from 2 to 10 percent, as noted in our footnote 3. Indeed, focusing in this range, most models manage to deliver values of the asymptotic speed within it. The point here, is that pretty much all reasonable models of economic growth deliver reasonable asymptotic speeds of convergence and it would be quite misguided to reach conclusions about the performance of the model without investigating the whole time path of the convergence speed as well as other important variables that may be changing along the transition.

Let us now concentrate on the relatively low asymptotic speed of convergence that we obtain in our multisectoral model. As a general rule, adding a state variable tends to slow down the speed. For example, Turnovsky (2004) introduces public capital into a non-scale version of the Ramsey model and reduces the asymptotic speed of convergence to around 2.3 percent, a value very close to ours. So, in light of previous literature, it is not entirely surprising that adding human capital into the non-scale R&D-based growth model reduces the convergence speed.

This last example help us to illustrate one last time the main point of our paper. Since 2.3 percent is pretty close to the values that we obtain for the asymptotic speed, should we conclude that the model with public capital studied by Turnovsky (2004) performs as well as our model with human capital? Our answer is clear: no! We can not discriminate between the two models unless we carefully investigate the whole adjustment path.

7 Conclusion

In this paper, we have compared transitional dynamics of two alternative non-scale R&D-based models of economic growth. One model incorporates human capital accumulation, whereas the other does not. We have shown that the asymptotic speed of convergence of per-worker output predicted by both models are consistent with empirical evidence. This might have led us to believe that the theory in which human capital and technology have an important complementary role does not represent an improvement over the theory that does not emphasize this role. However, this information given by the asymptotic speed of convergence is quite misleading because we also show that when we examine the entire adjustment path, the model with human capital offers significantly better predictions regarding the evolution of growth rates. This has led us to conclude that a model that delivers an asymptotic speed of convergence that complies better with empirical estimates does not necessarily provide a better description of the convergence process. A careful study of the adjustment paths of key variables predicted by alternative growth theories starting far away from the balanced growth path is required in order to successfully discriminate among them.

The paper offers another interesting insight. We have shown that the introduction of human capital makes the asymptotic speed of convergence much less sensitive to external shocks that affect policy parameters in the model. This is consistent with Barro and Sala-i-Martin's (1995) result that estimated convergence speeds do not vary much across different region groups that belong to developed nations. Our intuition for this result is that, as we increase the number of state variables, labor must be allocated among more sectors, thus reducing the speed at which they can converge towards the steady-state. But unlike the interpretation that the literature has assigned to Barro and Sala-i-Martin's finding, we can not conclude that policy actions have a small effect on the convergence speed, because non-scale growth frameworks deliver speeds of convergence that can vary over time.

Appendix

Data

The data and programs used in this paper are available by the authors upon request.

- *Income (GDP)* [Source: PWT 5.6]

Cross-country real GDP per worker (chain index, 1985 international prices) is taken from the Penn World Tables, Version 5.6 (PWT 5.6) as described in Summer and Heston (1991). This data set is available on-line at: http://datacentre.chass.utoronto.ca/pwt/index.html.

- *Physical capital stocks* [Source: STARS (World Bank), and PWT 5.6]

Physical capital comes from PWT 5.6. However, this dataset reports physical capital starting in 1965. To obtain stocks from 1963 for South Korea, and from 1960 for Japan, we used the growth rates implied by the STARS physical capital data to deflate the 1965 PWT 5.6 numbers.

- *Education* [Source: STARS (World Bank)]

Annual data on educational attainment are the sum of the average number of years of primary, secondary and tertiary education in labor force. These series were constructed from enrollment data using the perpetual inventory method, and they were adjusted for mortality, drop-out rates and grade repetition. For a detailed discussion on the sources and methodology used to build this dataset see Nehru, Swanson, and Dubey (1995).

Methodology

In what follows we present a brief explanation of the methodology used in analyzing transitional dynamics. Because there is no analytical solution to our system of Euler and motion equations, we resort to numerical approximation techniques. In our analysis we follow Judd (1992) to solve the dynamic equation system, approximating the policy functions employing high-degree polynomials in the state variables.

In particular, the parameters of the approximated decision rules are chosen to (approximately) satisfy the Euler equations over a number of points in the state space, using a nonlinear equation solver. A Chebyshev polynomial basis is used to construct the policy functions, and the zeros of the basis form the points at which the system is solved; that is, we use the method of orthogonal collocation to choose these points. Finally, tensor products of the state variables are employed in the polynomial representations.

This method has proven to be highly efficient in similar contexts. For example, in the one-sector growth model, Judd (1992) finds that the approximated values of the control variables disagree with the values delivered by the true policy functions by no more than one part in 10,000. All programs were written in GAUSS and are available by the authors upon request.

Table 6: Accuracy measures in different models

			Average Error (%)			Max. Error (%)		
Country	*Model**	ψ	C	u_H	u_A	C	u_H	u_A
Japan	*model with H*	0.131	0.01	0.02	0.01	0.04	0.07	0.04
Japan	*model w/o H*	0.10	0.00	−.−	0.00	0.01	−.−	0.02
S. Korea	*model with H*	0.162	0.06	0.17	0.06	0.27	0.78	0.24
S. Korea	*model w/o H*	0.074	0.01	−.−	0.01	0.02	−.−	0.05

* *model with H* refers to the per worker three-sector non-scale growth model with schooling sector. *model w/o H* refers to the two-sector non-scale growth model without schooling sector.

For the cases considered in this paper, Table 6 gives accuracy measures. In particular, we assess the Euler equation residuals over 10,000 state-space points using the approximated rules. For each variable, the measures give the average and maximum current-value decision error that agents using the approximated rules make, assuming that the (true) optimal decisions were made in the previous period. Santos (2000) shows that the residuals are of the same order of magnitude as the policy function approximation error.

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