The Public and Private MPK*

Matt Lowe
MIT
mlowe@mit.edu

Chris Papageorgiou
IMF
cpapageorgiou@imf.org

Fidel Perez-Sebastian
U. Alicante and U. Hull
fpse2c@gmail.com

March 2014

Abstract

Why doesn't capital flow to developing countries as predicted by the neoclassical model? Is the explanation simply that the cross-country marginal productivity of capital (MPK) is equalized, and if so, why? We revisit these issues by unpacking the MPK into its public and private components, since there is good reason to believe that the process of MPK determination is enormously different across the two sectors, especially in developing countries. We do so by calculating MPK schedules for the two sectors in a large sample of advanced and developing countries. The main findings are twofold: using updated investment data shows that the MPK is not only flat but rather slightly positively sloped. More importantly, this finding is mainly driven by the public sector – the public MPK is strongly positively sloped whilst the private MPK is flat. We offer an intepretation of this surprising result and advance new explanations for the Lucas Paradox related to the behavior of the public sector.

JEL Classification: O41, O47

Keywords: Marginal product of public and private capital, public sector inefficiencies, capital flows, the Lucas Paradox.

*Special thanks go to Francesco Caselli for detailed comments. We also benefited from discussion with Andy Berg, Steve Bond, Bob Chirinko, Paul Collier, Raphael Espinoza, Jim Feyrer, Alvar Kangur, Aart Kraay, Robert Lucas, Era Dabla-Norris, Cathy Pattillo, Francis Teal, Rick Van der Ploeg, Tony Venables and seminar participants at the IMF, Aberdeen, Birmingham, Oxford, Royal Holloway University of London, the Conference on Economic Development and Growth, Barcelona 2011, the CSAE conference, Oxford 2012, and the AEA Meetings, San Diego 2013. Matt Lowe gratefully acknowledges financial support from ESRC, award number ES/I02476X/1, and Perez-Sebastian from the Spanish Ministry of Science and Technology, SEJ 2007-62656, FEDER funds, and by the Instituto Valenciano de Investigaciones Económicas. The views expressed in this study are the sole responsibility of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management.

1 Introduction

If capital-labor ratios are low in poor countries and returns high as the standard one-sector growth model predicts, why doesn't more capital flow from rich to poor countries? This fundamental question known as the Lucas Paradox, coined after Lucas' (1990) seminal paper, is a focal point for many key areas of economic development; whether the efficacy of aid, the extent of international capital market frictions, or the importance of institutions and complementary factors.

The paradox pre-supposes a downward-sloping financial return to investment in the cross-section of nations. Intuitively, there are only a small set of possible explanations: (i) either the return has been mis-measured, and it is not actually downward-sloping; (ii) the return is downward-sloping but capital movement is restricted by capital market imperfections; or (iii) investors in some way defy standard theories of profit maximization (e.g. there is some home bias which permits returns differentials to persist). Lucas himself posited that the explanation could be that of failing to account for complementary factors to physical capital, such as human capital, resulting in an overstating of the MPK. Lucas placed little credence on the argument of capital market frictions.[1]

The aggregate MPK is the most common measure employed to approximate the return to investment, in an attempt to resolve the paradox. Unfortunately estimating the MPK is no easy task. Several approaches exist: among them, comparison of interest rates across countries, production function estimation, and calibration.[2] Caselli and Feyrer (2007) (CF from hereafter) argue that these approaches have collectively failed at producing reliable and comparable estimates of the cross-country MPK.[3] In a persuasive, yet provocative, contribution to the literature, they present the case for direct MPK estimation using easily accessible macroeconomic data.[4] Their approach assumes competitive markets and imposes no restrictions on production functions other than that of constant returns to scale.

CF's main contribution is that they derive an MPK measure that is more suitable for the purpose

[1] In a recent contribution, Gourinchas and Jeanne (2011) argue that international capital flows more towards developing countries with lower (not higher) productivity growth. While the Lucas Paradox is about the small magnitude of capital inflows to developing countries the "allocation puzzle", as coined by Gourinchas and Jeanne, is about the allocation of the already small size of capital flows across developing countries.

[2] See Banerjee and Duflo (2005) for an extensive review.

[3] According to CF, the comparison of interest rates in developing countries is problematic because markets are heavily distorted, identification in production function estimation presents many caveats, and calibration exercises rely on specific forms of the production function.

[4] Taylor (1998) measures the MPK similarly for Argentina, and Bai *et al.* (2006) use a similar approach to measure the return to capital (both in aggregate and by sector/region) in China, though they use current price data to measure $P_Y Y / P_K K$ rather than real data followed by a price adjustment as in CF.

of international credit flows. The standard MPK derived from the one-sector growth model is not a good measure of capital returns because it provides output per unit of physical capital invested. Multisector models imply that the financial return to investment is better proxied as output per unit of output invested. By making two reasonable adjustments to a naïve measure of the MPK, Lucas Paradox resolved, CF find that the cross-country MPK is roughly flat. Yet CF's measure is based on total capital, whereas the relevant MPK for investors is the return to private capital.

In this paper, we attempt to go beyond CF and dig deeper into their resolution of the Lucas Paradox by distinguishing between the public and private MPK.[5] The private and public distinction is important for a host of additonal reasons. First and foremost amongst these is that the theory behind MPK determination is likely to differ significantly between the two sectors. There is much literature elsewhere with results that hinge on the contrasting behavioral idiosyncrasies of public and private agents (e.g., Becker 1957, Fama 1980, Besley and Burgess 2002, Robinson and Torvik 2005).[6] The empirical evidence in Keefer and Knack (2007) is also consistent with the notion of governments as non-maximizers. To be consistent with MPK equalization, public investment should be highest where the returns are highest. Keefer and Knack find instead that public investment is dramatically higher in countries with low-quality governance and limited political checks and balances. Their interpretation of this result is that governments use public investment as a means for rent-seeking. If the public sector maximizes an entirely different objective function to the private sector, capital allocation and the resulting MPK should be determined differently.[7]

Second, the private and public sectors tend to make different types of investment. The public sector tends to invest where markets fail: where social returns exceed private returns, where the capital is non-rivalrous and non-excludable and where high fixed costs make a natural monopoly a strong possibility. In short, public and private capital should be considered imperfect substitutes in a country's production function (CF implicitly considered them to be perfect substitutes). In this sense, the overall MPK is misleading, whilst the private and public MPKs are more informative.

[5] This within-country heterogeneity in returns is a key theme in Banerjee and Duflo (2005).

[6] Robinson and Torvik (2005), for example, aim to explain why governments don't act like profit maximizers when it comes to investing. In particular, the model explains the political motivation behind the construction of white elephants. Politicians construct these inefficient projects when they find it difficult to make credible promises to political supporters. The general point of this and other political economy models is that governments are driven more by an electoral motive than by a profit motive.

[7] This is particularly true when we consider public capital as capital that the private sector is unwilling or unable to provide given difficulties in converting the benefits into a stream of profits or given government regulations. In this scenario, public MPK differences can persist since private actors cannot (or will not) step in to exploit arbitrage opportunities.

Third, following Pritchett (2000), the separation between public and private capital is warranted in light of public investment inefficiency. As Pritchett emphasizes, there is no plausible behavioral model by which we would expect public investment to be efficient in the same way that might be expected of private investment. Caselli (2005) echoes this sentiment and argues for the future separation of public and private investment, when appropriate data comes available, in the context of development accounting. In this paper, we use data that makes this separation possible.

Pritchett (2000) and Caselli (2005), among others, thought that data issues would make it near impossible to convincingly estimate the private and public MPK. One main contribution of this paper is to break the impasse and carry out this exercise for the first time. To do this, we employ improved data on the sectoral share of investment from IMF's World Economic Outlook (WEO) to estimate the private and public MPK for a broad sample of advanced and developing countries.

Our main finding is surprising: the overall MPK is not flat but rather somewhat positively sloped. Most importantly, this is largely driven by the strongly positively sloped public MPK – the private MPK is flat. This finding is subsequently explored by several extensions and an extensive robustness analysis pointing to public sector frictions rather than financial frictions as a key constraint to enhancing the MPK and with it, accelerating international capital inflows. This distinct behavior of the public sector sheds new light on the formation of capital in developing countries and promotes new explanations of Lucas Paradox.

The analysis yields two other important results. First, in all nations, but especially in developed countries, the marginal productivity of public capital varies much more than its private counterpart, indicating potentially greater misallocation in public than in private capital. Second, efficiency requires that investment returns across the public and private sectors should be equalized; we find, however, significant dispersion around one in the ratio of public to private marginal productivity of capital, with about half less-developed countries above and below this threshold. Most developed countries, on the other side, exhibit underinvestment in public investment.

We proceed as follows. Section 2 takes a close look at the primary sources of the data used to disaggregate total capital into its public and private components and discusses the steps followed to calculate the public and private MPK. Section 3 presents and discusses the new trends unravelled from the data disaggregation. Section 4 extends the main analysis in several directions, including incorporating inefficiencies in the measurement of the MPK, extending the dataset and a number of robustness checks. Section 5 concludes.

2 Data

In this section we show in detail the steps followed to construct the public and private marginal product of capital. First of all, we derive the MPK measures. Following CF, suppose that there are J final goods in the economy, including capital and consumption products. In any one of these sectors, let us say sector i, production occurs using private capital (K_{pi}), public capital (K_{gi}), and other inputs (X_i) according to $Y_i = F(K_{pi}, K_{gi}, X_i)$. The role of public capital in F could be indirect, if the public sector provides a flow of services to the private economy, instead of a stock of capital. In this case, government's output would represent an intermediate input, and we could think of the above production function as a reduced form.

Assume as well that F displays constant returns to scale, and there is perfect competition in all markets. Constant returns imply that we abstract from possible externalities from public capital; or more precisely, from the possibility that the variable that belongs to the production function is K_g rather than K_{gi}, where K_g denotes the economy's total stock of public capital. As we will discuss in Section 4, the constant returns assumption is consistent with most available evidence. Perfect competition, in turn, might not constitute a good representation of market structure, especially in LICs. In Section 4, we will also seek to address this issue by using an alternative measure that is immune to both assumptions.

Abstracting from capital gains, the return to one unit of income invested in private and public capital employed by sector i equals, respectively:

$$\frac{P_i \ MPKP_i}{P_{k_p}}, \qquad \text{and} \qquad \frac{P_i \ MPKG_i}{P_{k_g}}; \tag{1}$$

where P_i is the price of good i; $MPKP_i$ and $MPKG_i$ represent the marginal product of private and public capital in sector i, respectively; and P_{k_p} and P_{k_g} give the price of private and public capital.

Total income attributed to each of the two capital types should then equal: $\sum_{j=1}^{J} P_j MPKP_j K_{pj}$, and $\sum_{j=1}^{J} P_j MPKG_j K_{gj}$. If the available amounts of private and public capital are allocated efficiently among sectors, it must hold that $P_j MPKP_j = P_i MPKP_i$, and $P_j MPKG_j = P_i MPKG_i$, for all i and j. Hence, we can write capital income derived from its private and public components as: $P_i MPKP_i K_p$ and $P_i MPKG_i K_g$, respectively; where $K_g = \sum_{j=1}^{J} K_{gj}$ is the stock of public capital; and $K_p = \sum_{j=1}^{J} K_{pj}$ is the stock of private capital. Finally, let us use β to denote the share

of private capital in national income, and γ to denote the one of public capital. They equal

$$\beta = \frac{P_i MPKP_i K_p}{P_y Y}, \qquad \text{and} \qquad \gamma = \frac{P_i MPKG_i K_g}{P_y Y}; \tag{2}$$

where $P_y Y$ is the value of GDP. Expression (2) implies that the return given by (1) is the same across final-good sectors; as a consequence, the return to investment in private and public capital (denote them by $MPKP$ and $MPKG$, respectively) can be computed as:

$$MPKP = \beta \frac{P_y Y}{P_{k_p} K_p}; \qquad MPKG = \gamma \frac{P_y Y}{P_{k_g} K_g}. \tag{3}$$

These two last measures constitute our relevant price-corrected MPKs.[8]

The assumptions made in the last paragraph work well for the private sector. However, they may not be fully appropriate for the public sector. For instance, when the government provides services, it may not be equalizing returns among industrial activities or across regions (e.g., through ethnic favouritism).[9] In addition, it is not straightforward how to determine the share of income that should be attributed to public capital in national accounts. As a consequence, we will use production function estimates to help approach the contribution of the two types of capital, and conduct extensive robustness analysis. Another implication is that our findings related to the public MPK should be interpreted with more caution.

Our core sample comprises 50 developing and developed countries with public, private and overall MPK data in 2006.[10] We also look at time series data from 1990, with the sample size beginning at 52, but falling to 48 across 1990-2009. As in CF, the main constraint on sample size is due to the need for data on the overall capital share taken from Bernanke and Gurkaynak (2001). In Section 4.2 we use updated data to relax this constraint, enabling us to extend the sample initially to 74 countries, then with some assumptions, to 130 countries.

We measure the cross-country private and public MPK using current price local currency data from World Development Indicators (WDI), rather than real data from Penn World Tables (PWT) adjusted for relative price differences as in CF. The use of current price local currency data is

[8] Appendix 1 offers a full-blown partial equilibrium three-sector growth model that allows for public investment inefficiency, and examines how such a public sector distortion may affect our core measures. In addition, we explore conditions under which the public and private MPK can obtain the differential schedules delivered by the empirical analysis in section 3.

[9] Notice that even if the government equalizes marginal productivities across sectors, the overall level of public capital provision may not be the efficient one to equalize the public MPK to the private one. However, our measure is not affected by this second type of public sector inefficiency; in fact, our paper tries to quantify it.

[10] Though we have data for subsequent years until 2010 (sample sizes of 49, 49, 48 and 23 in 2007, 2008, 2009 and 2010, respectively).

preferred here since it side-steps any reliance on PPP adjustments and extrapolated ICP data shown to be quite unstable for non-OECD countries (see Johnson *et al.,* 2013). In addition, it has been argued elsewhere (e.g. Knowles, 2001) that investment shares are more accurately measured using local price data, rather than data from PWT. In any case, for our analysis the two approaches yield essentially the same results, as will be seen in Section 4. The data we require are: income shares of public and private capital (β, γ), GDP in current price local currency $(P_y Y)$, public capital $(P_{k_p} K_g)$ and private capital $(P_{k_g} K_p)$.

Current price local currency data on GDP and investment are taken from WDI. In principle, each capital series could be obtained by using the perpetual inventory method on current price historic investment data, deflated each year by a sector-specific investment deflator. In practice however, only a common investment deflator exists.[11] In applying this common deflator, we are constrained in identifying differences in the relative price of capital faced by the public and private sector. But for the baseline estimation, the public and private MPK are adjusted by the same price ratio.[12]

With current and constant price investment data, the next step is to split these investment flows into their private and public sector constituents. This split is crucial as it drives the resulting differences in the private and public MPK. To do this disaggregation, we use private investment share data from the World Economic Outlook (WEO). For the sample of 50 countries in 2006, the mean number of time series observations of the private investment share is 33 (ranging from a minimum of 12 to a maximum of 49).[13] Before total investment is disaggregated, the first available observation of the private investment share is extrapolated back to the first year of investment data.

In the absence of any investment data at all prior to 1960, it is necessary to set initial conditions for both the public and private capital stocks. As is common practice (given the notion of a steady-state capital stock), we set the initial condition, K_{j0}, to $I_{j0}/(\mathfrak{g}_j + \delta_{j0})$ where private and public sectors are indexed by $j = p, g$. I_{j0} is current price investment in the first year available, $\mathfrak{g}_j$ is the

[11] This investment deflator is derived from WDI data as 100*(current price local currency gross fixed capital formation/constant price local currency gross fixed capital formation). Missing constant investment data is set equal to the product of constant price GDP and gross fixed capital formation as a proportion of GDP, for countries with available data.

[12] In Section 4.4 we relax this assumption by using a proxy variable for the relative price of private to public investment. Our headlines are unchanged.

[13] For seven countries (Austria, Denmark, Spain, Greece, Ireland, Jamaica and Sweden) investment share data was missing from the latest WEO. We opted to take the share data from WEO 2003, using forecasted shares for the years 2004-2008.

country- and sector-specific average growth rate of constant price investment over the first twenty years of available data,[14] δ_{j0} is the relevant depreciation rate for the first year of available investment data.[15] Caselli (2005) shows that sufficiently recent capital measures tend to be insensitive to the exact assumptions made on these initial conditions. Armed with disaggregated investment and deflator data, assumptions on initial conditions and a pattern of depreciation rates, we apply the perpetual inventory method to construct current price capital series for each country as follows:

$$P_{kt}K_{jt} = (1-\delta_{jt})\left(\frac{P_{kt}}{P_{kt-1}}\right)P_{kt-1}K_{jt-1} + I_{jt-1}, \tag{4}$$

so that

$$P_{kt}K_{jt} = (1-\delta)^t\left(\frac{P_{kt}}{P_{k0}}\right)\frac{I_{j0}}{\mathfrak{g}_j+\delta_{j0}} + \sum_{i=1}^{t}(1-\delta)^{t-i}\left(\frac{P_{kt}}{P_{k0}}\right)I_{ji-1}. \tag{5}$$

The total capital stock is then simply set equal to the sum of the private and public stocks. The capital measures become less sensitive to the initial conditions and investment share extrapolation as t becomes closer to the present. This suggests that time series results have to be interpreted with greater care than the cross-section results in 2006.

Having constructed public and private capital stocks, the remaining specification choice is that for income shares β and γ. Unlike the share of capital in income, these shares cannot be straightforwardly derived from national accounts data. We proceed instead by taking the overall share of reproducible capital, $\beta+\gamma$, to be equal to the share data used by CF (α_k). This share data derives initially from Bernanke and Gurkaynak (2001).[16] CF then make an additional adjustment using wealth data from World Bank (2006) in order to account for natural capital. The result is data on the share of reproducible capital in income, α_k.

Next, we take the composition of this reproducible capital share to be consistent with the results of Gupta et al. (2011). Their approach is to estimate system-GMM panel regressions assuming a Cobb-Douglas production function with skill-adjusted labor, private and public capital

[14] Since a negative g_j could result in implausibly large or impossibly negative initial conditions, the measure was bounded at zero. For the core sample of fifty countries, this bounding only affected the public capital initial condition for Zambia.

[15] The depreciation rates employed are as follows (all 1960-2008): (1) Public sector: 2.5% in LICs, 2.5% rising to 3.4% in MICs and 2.5% rising to 4.3% in Advanced; (2) Private sector: 4.25% in LICs, 4.25% rising to 7.6% in MICs, 4.25% rising to 9.6% in Advanced. We extrapolated the 2008 income- and sector-specific depreciation rates to 2009 and 2010. Alternatives to this baseline assumption did not change the results qualitatively.

[16] Bernanke and Gurkaynak (2001) themselves extended an earlier dataset compiled in Gollin (2002). We extend their data still further in Section 4.2.

as its arguments. Since the estimation is in logs, each coefficient captures the income share of the associated factor input.[17] Still, we don't take the absolute coefficients for our measurement since the aim is to maintain full country-specificity of the reproducible capital share in income. In addition to this, we place more credence on shares derived from national accounts as opposed to those derived from regression estimates. With this in mind, we use the results in Gupta et al. (2011) (see columns (2-3) in their Table 6) to infer only the relative income shares of public and private capital for income groups, with $\beta/(\beta+\gamma) = 0.63$ in Middle Income and Advanced economies, and $\beta/(\beta+\gamma) = 0.48$ in Low Income Countries (LICs) (i.e. the relative income share of public capital is lower in richer countries).

This is certainly an imperfect approach to measuring income shares for two reasons: first, there are concerns about identification of growth estimates; second, we lack full heterogeneity in relative shares across countries. On the first issue at least, while concerns about identification are warranted, extensive robustness analysis suggests that results are unaltered qualitatively under a large number of alternative estimated income shares. In other words, a wide range of economically plausible income shares leave the main results unchanged. Robustness of the main results is also supported by a counterfactual exercise discussed below in Section 4 - in this exercise we see how much insight we can get by ignoring measured income shares entirely. We argue that taken together, the data excluding income shares remains consistent with our main claims.

3 Public and Private MPK Calculations

With the necessary data at hand we turn to calculating each country's private and public MPK. Table 1 presents baseline summary statistics for 2006, unless stated otherwise. It is reassuring that the only difference between our country sample and that of CF is the loss of Jamaica and Trinidad and Tobago due to a lack of recent investment data covering these countries.

Next we attempt to reproduce CF's main results on the overall MPK using our baseline dataset. Let Y and P_y be the quantity and price of final goods; K and P_k the quantity and price of capital goods; α_w the share of reproducible *plus* natural capital; and α_k the share of reproducible capital. By construction, $\alpha_w > \alpha_k$. CF define the following MPK measures:

$$MPKN = \alpha_w \frac{Y}{K}; \quad MPKL = \alpha_k \frac{Y}{K}; \quad PMPKN = \alpha_w \frac{P_y Y}{P_k K}; \quad PMPKL = \alpha_k \frac{P_y Y}{P_k K}.$$

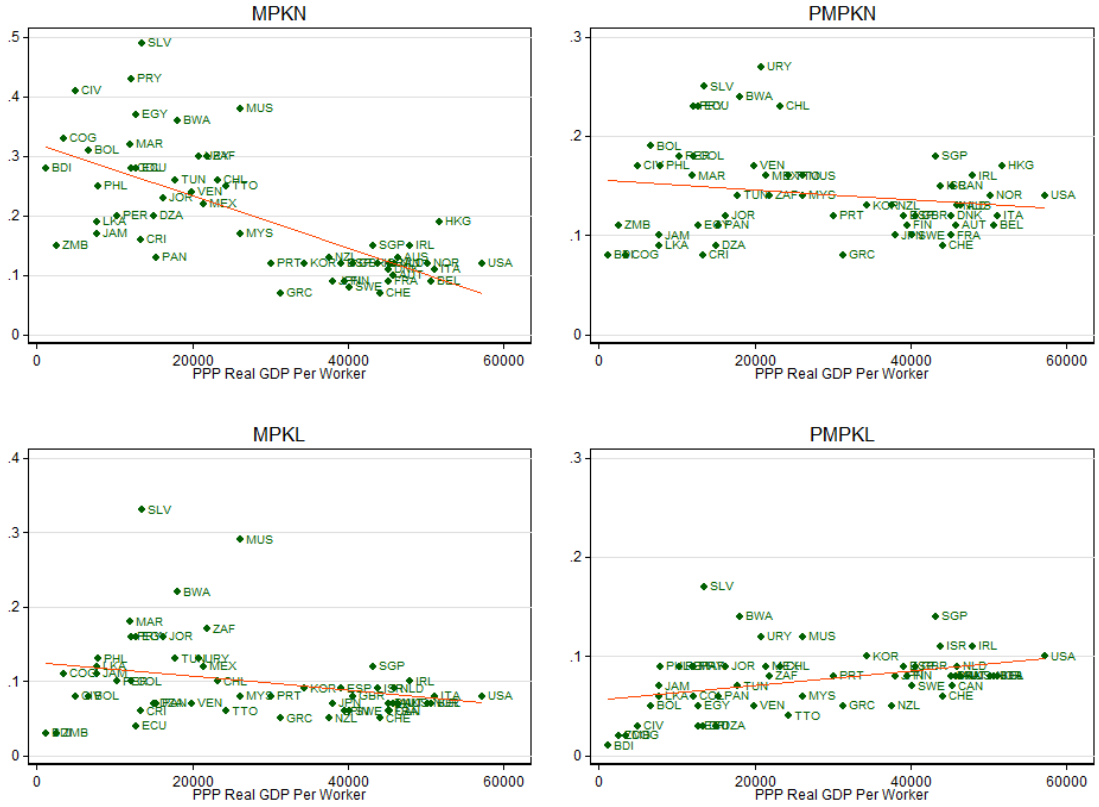
[17] Important to note that the shares estimated by Gupta et al. (2011) are consistent with constant returns to scale over labor, private capital, and public capital.

Table 1: Summary Statistics

Country	ISO	y	Sh#	MPK(96)	MPK	MPKG	MPKP
Algeria	DZA	14560.1	21	0.03	0.05	0.04	0.06
Australia	AUS	75219.6	23	0.08	0.09	0.12	0.08
Austria	AUT	73793.3	40	0.08	0.09	0.20	0.07
Belgium	BEL	77973.8	16	0.08	0.10	0.25	0.07
Bolivia	BOL	7933.45	30	0.05	0.04	0.05	0.03
Botswana	BWA	19477.6	31	0.14	0.16	0.12	0.19
Burundi	BDI	702.85	31	0.01	0.01	0.01	0.02
Canada	CAN	68609.3	32	0.07	0.09	0.17	0.07
Chile	CHL	27516.7	26	0.09	0.11	0.25	0.09
Colombia	COL	17441	42	0.06	0.07	0.06	0.08
Congo, Republic of	COG	5138.94	42	0.02	0.05	0.10	0.04
Costa Rica	CRI	23514.9	20	0.03	0.07	0.10	0.06
Cote d'Ivoire	CIV	3352.81	41	0.03	0.02	0.02	0.02
Denmark	DNK	67132	31	0.08	0.11	0.24	0.08
Ecuador	ECU	14134.6	32	0.03	0.03	0.02	0.04
Egypt	EGY	13872.9	42	0.05	0.05	0.04	0.06
El Salvador	SLV	15734.4	30	0.17	0.16	0.16	0.16
Finland	FIN	65269	12	0.08	0.10	0.17	0.08
France	FRA	68105.4	15	0.08	0.09	0.13	0.08
Greece	GRC	61528.3	40	0.05	0.08	0.07	0.08
Ireland	IRL	75376.3	49	0.11	0.10	0.13	0.09
Israel	ISR	60113.8	27	0.11	0.14	0.27	0.11
Italy	ITA	70881.8	23	0.08	0.10	0.18	0.08
Jamaica	JAM	20282.8	40	0.07			
Japan	JPN	64150.5	31	0.08	0.09	0.09	0.09
Jordan	JOR	14078.7	48	0.09	0.10	0.08	0.11
Korea, Republic of	KOR	48077.8	41	0.1	0.12	0.19	0.09
Malaysia	MYS	25977.6	42	0.06	0.07	0.05	0.10
Mauritius	MUS	19235.3	38	0.12	0.15	0.12	0.18
Mexico	MEX	30136.5	38	0.09	0.14	0.20	0.12
Morocco	MAR	8860.36	41	0.09	0.10	0.19	0.08
Netherlands	NLD	75013.1	31	0.09	0.11	0.16	0.10
New Zealand	NZL	52261.3	26	0.05	0.06	0.06	0.05
Norway	NOR	94797.3	21	0.08	0.13	0.19	0.11
Panama	PAN	18734.6	42	0.06	0.09	0.11	0.08
Paraguay	PRY	7943.12	42	0.09	0.08	0.07	0.08
Peru	PER	13645	42	0.09	0.14	0.21	0.12
Philippines	PHL	6540.46	25	0.09	0.10	0.14	0.09
Portugal	PRT	38554.6	16	0.08	0.08	0.14	0.07
Singapore	SGP	84558.7	32	0.14	0.20	0.25	0.17
South Africa	ZAF	19752.7	40	0.08	0.13	0.19	0.11
Spain	ESP	58428.5	40	0.09	0.10	0.17	0.08
Sri Lanka	LKA	8347.63	34	0.06	0.08	0.14	0.06
Sweden	SWE	70425.8	49	0.07	0.09	0.13	0.08
Switzerland	CHE	64887.2	31	0.06	0.08	0.14	0.06
Trinidad and Tobago	TTO	45789.9	36	0.04			
Tunisia	TUN	15714.4	36	0.07	0.08	0.14	0.07
United Kingdom	GBR	68307.3	32	0.09	0.11	0.15	0.09
United States	USA	84597.9	32	0.1	0.10	0.16	0.09
Uruguay	URY	19200	42	0.12	0.08	0.06	0.10
Venezuela	VEN	20038.2	43	0.05	0.07	0.04	0.12
Zambia	ZMB	3942.16	31	0.02	0.01	0.00	0.01

Note: ISO refers to each country's isocode, y is PPP Real GDP Per Worker from PWT 7, Sh# indicates the number of time series observations of sectoral investment shares, MPK(96) refers to CF's MPK measure for 1996 using PWT data.

Figure 1: Main CF Results



$MPKN$ is the naïve MPK, while $MPKL$ adjusts the income share of capital to exclude natural capital, and $PMPKN$ controls for the relative price of final goods to capital products. Finally, we arrive at the preferred measure, $PMPKL$, which incorporates natural-capital with relative-output-to-capital-price. Table 2 presents summary statistics of CF's four main MPK measures with increasing sophistication for 1996. CF argue on the basis of the final measure (PMPKL) that the MPK is essentially flat.

CF's results are also illustrated in Figure 1. Fitted lines are added to obtain a better sense of the implied relationship between the MPK measures and income levels. The top-left panel in Figure 1 shows clearly that the naïve MPK implied by the standard neoclassical one-sector model is downward sloping. As we move to the right or down, each of the two adjustments disproportionately reduces the MPK in developing countries, since developing countries tend to have a higher share of

Table 2: Caselli and Feyrer (2007) MPK Measures

Measure	Mean	Std. Dev	Min	Max
MPKN	0.20	0.10	0.07	0.49
PMPKN	0.14	0.05	0.08	0.27
MPKL	0.10	0.06	0.03	0.33
PMPKL	0.08	0.03	0.01	0.17

natural capital in income and tend to face a higher relative price for capital goods.[18] The preferred MPK measure, $PMPKL$ (bottom-right panel), is actually slightly upward sloping (the fitted line is in fact statistically significant at the one percentage level, though its economic significance could be disputed). CF's main result then suggests that international capital markets do a good job of allocating capital efficiently across countries, and that there are no downward financial frictions in capital movements.[19] Provocative as it is, there is, in our view, not much cause to argue with the result presented in CF. Both adjustments are reasonable and make intuitive sense. Whilst the individual country MPK estimates could be challenged on the basis of data, the overall shape of the MPK schedule is quite robust.[20]

Closer inspection of the charts demonstrates that this result obtains mostly from CF's adjustments affecting developing, not advanced economies. Focusing our attention on advanced economies, with Portugal (PRT) the poorest of this group, we can see that the naïve MPK measure ($MPKN$) is slightly increasing even without the two adjustments made in CF. The main finding of a non-decreasing MPK obtains from the impact of the adjustments on developing economies. Before the adjustment, there is no clear relationship between the return and income in developing nations, whereas after it a clear non-negative relationship is unravelled.

The core analysis in the remainder of the paper focuses on shedding light on these intriguing results by turning attention to the distinction between private and public capital. Put differently, the analysis will attempt to unpack the results found by CF by examining the role of the public sector

[18] Hsieh and Klenow (2003) point to the relatively low productivity in capital goods producing sectors in developing countries as a cause of the high relative price.

[19] For example, as CF show, the misallocation of capital implies an overall efficiency loss of only 0.1% of global GDP.

[20] Having said that, other adjustments can be suggested which in principle have the ability to overturn the result of a flat MPK. Chirinko and Mallick (2008) draw attention to the role played by adjustment costs, finding that a large MPK differential re-emerges once adjustment costs are accounted for.

Figure 2: The Composition of Investment

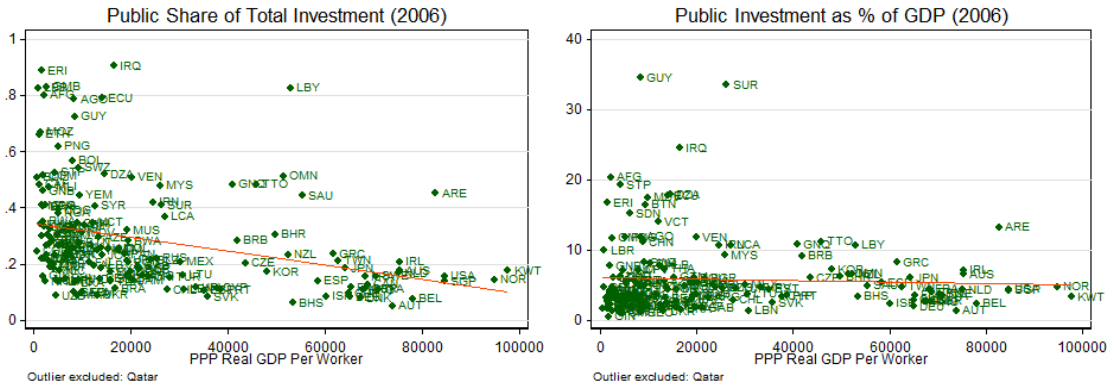


Table 3: The Composition of Public Investment

Variable	LICs*	MICs†	Advanced
Mean Public Share in Total Investment (2010)	42.6%	29.4%	16.6%
Mean Public Investment as % of GDP (2007)	6.0%	6.9%	3.9%

Source: World Economic Outlook, Penn World Tables

*LICs: low income countries

†MICs: middle income countries

in capital allocation. Our key motivation stems from the observed variation of public investment across nations. Specifically, Table 3 and Figure 2 show that the public sector plays a disproportionately large role in investment in developing countries compared to advanced economies.[21] Therefore disaggregating the MPK into private and public may have important implications for the slope of the overall MPK.

As explained in Section 2, our approach is to measure the cross-country MPK (overall, public and private) using current price data on income and capital along with income share data. Using current price data, we can estimate the MPK schedule for the exact same sample as CF, for the year 1996. This exercise is performed in Figure 3, which shows that the current price approach yields essentially the same overall MPK as CF, and that little has changed over the 10-year period 1996-2006. The advanced economies remain bunched closely around a financial rate of return of ten percent while the developing nations have a similar mean but greater variation, with rates of

[21] The relationship is flat when public investment as a percentage of GDP is considered because of two opposing forces: whilst public investment as a proportion of overall investment falls in income, investment as a proportion of GDP rises in income.

Figure 3: CF Results with Current Price Data

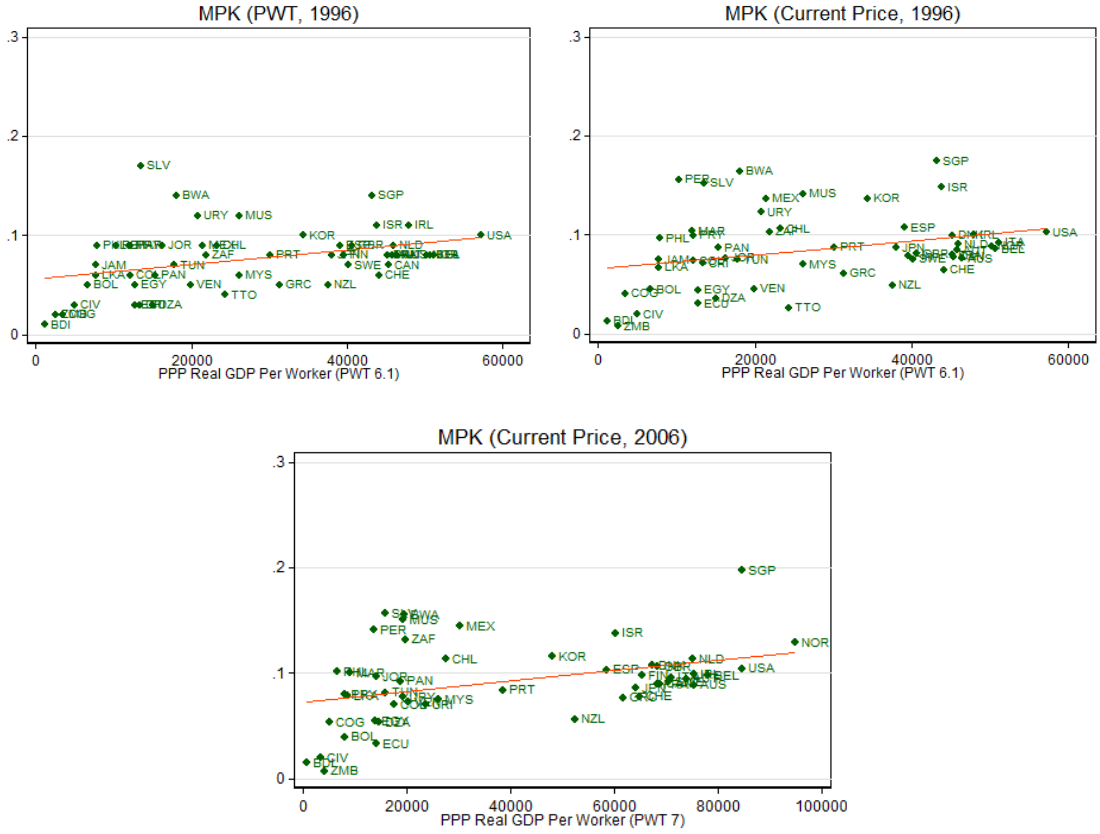


Table 4: Current Price Public and Private MPK

Measure	Year	Mean	Std. Dev	Min	Max
$MPKG$	1996	0.116	0.067	0.005	0.302
$MPKP$	1996	0.081	0.042	0.015	0.202
$MPKG/MPKP$	1996	1.525	0.813	0.240	3.596
$MPKG$	2006	0.130	0.069	0.005	0.265
$MPKP$	2006	0.085	0.037	0.014	0.193
$MPKG/MPKP$	2006	1.576	0.774	0.338	3.391

return from below one to sixteen percent.

Charts contained in Figure 4 show the public MPK ($MPKG$), its private counterpart ($MPKP$), and their ratio ($MPKG/MPKP$) for 1996 and 2006. Table 4 shows respective summary statistics. What jumps out is the distinctly different schedules between public and private MPKs.

Three observations are particularly notable. First and most important, the public MPK is upward sloping whereas the private MPK is flat. The fitted lines for the $MPKG$ measure (first row) are highly significant, whereas the ones for the $MPKP$ measure (second row) are insignificant. In fact, the evidence here suggests that it is the returns in the public sector which make CF's preferred overall MPK schedule slightly upward sloping. Once the public and private components of capital are separated, the return to investment relevant for markets, that is, the private MPK, suggest neither downward nor upward rigidities to international capital flows (though this suggestion is challenged slightly when we come to extend the sample). An upward sloping public MPK, on the other hand, fits with political economy stories such as Robinson and Torvik (2005) and Keefer and Knack (2007) of public sector inefficiencies.

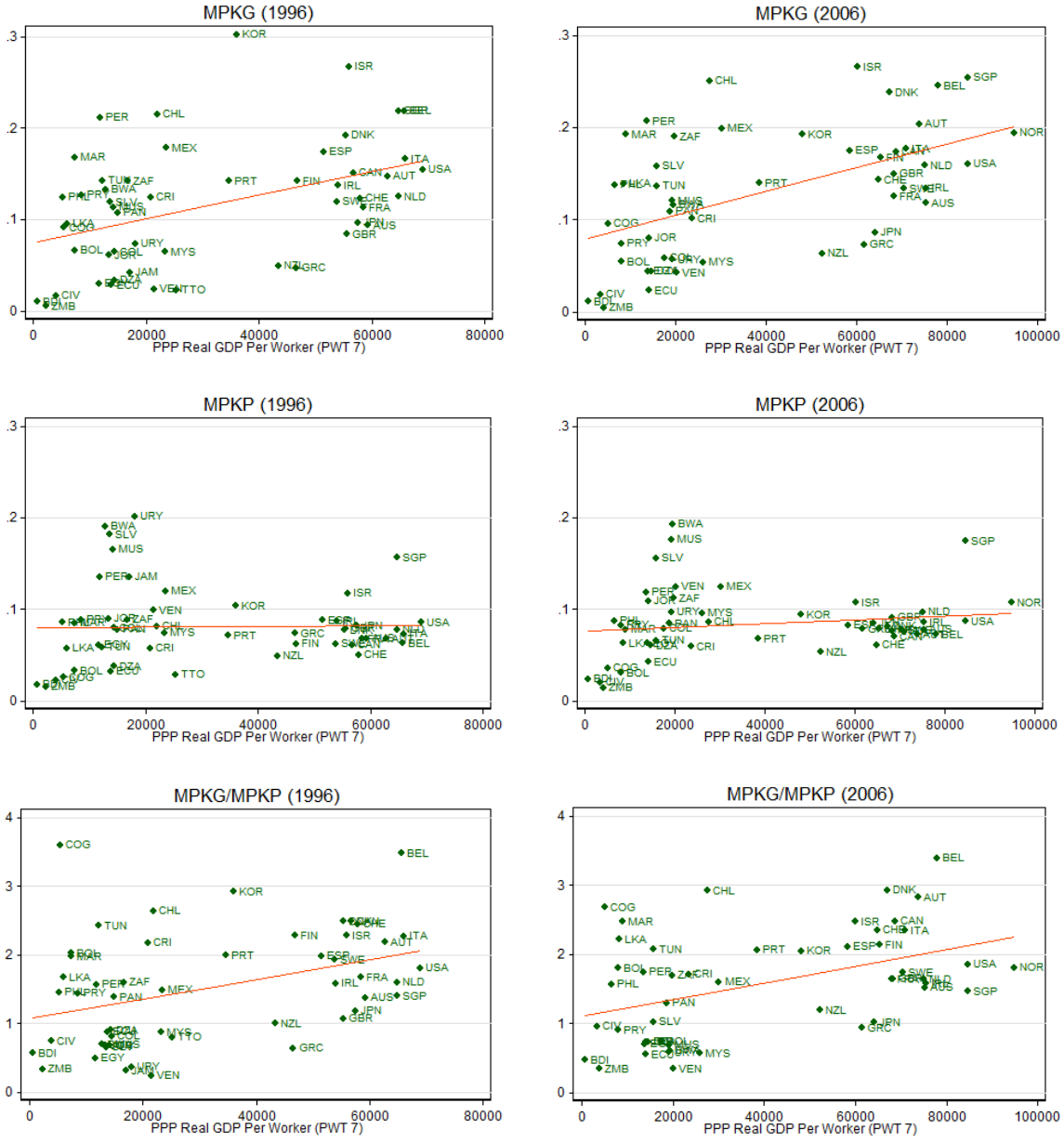
Second, there is greater variation in the public MPK. This indicates that the most significant loss in world GDP may be due to the misallocation of public capital, not private capital.[22]

Third, there is an interesting pattern amongst the advanced economies: the private MPKs are extremely similar, the public MPKs are much more dispersed, and the mean public MPK is higher. According to this, the graphs potentially tell another story: private capital is allocated efficiently in advanced economies, but inefficiently in poor countries; public capital is allocated inefficiently everywhere.

Efficiency requires not only that marginal returns are equalized across countries, but also across sectors. The ratio of the public to the private return, appearing in the third row of Figure 4,

[22] We show this to be the case more formally in Section 5.3.

Figure 4: Public and Private MPKs

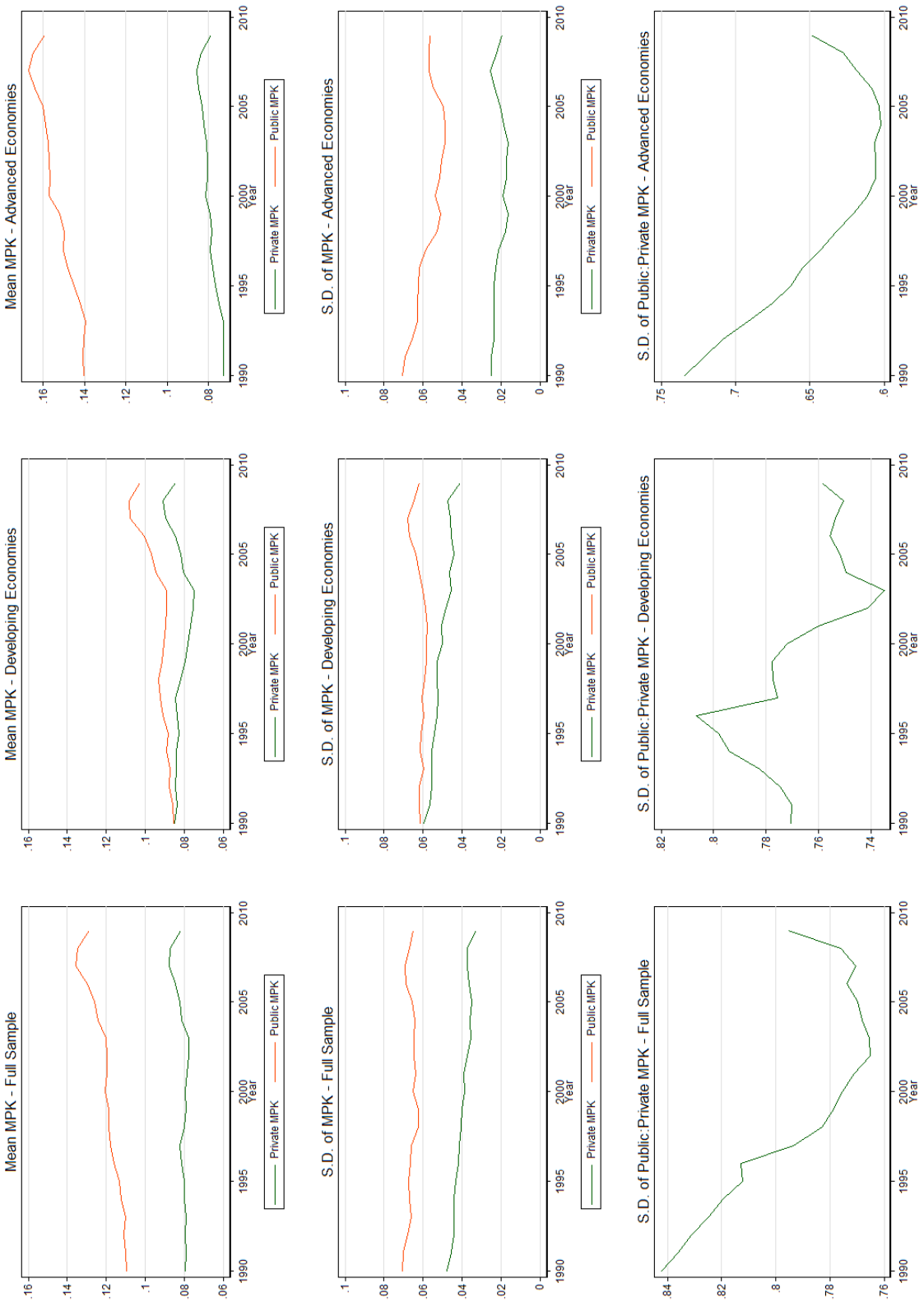


gives information about how countries deviate from this cross-sector equalization. The natural interpretation (given a benchmark that the private sector behaves optimally) is that a ratio below one reflects a government that overinvests in public capital, whereas a number above one suggests underinvestment. Few nations show values around one – the degree of dispersion is high. In 2006, 13 out of 27 developing countries provide a ratio above one, making the case for underinvestment in public capital. The developing country sample is then fairly evenly split between over and underinvestment in public capital, within-country, according to this measure. Governments in advanced economies, in contrast, appear consistently to accumulate too little public capital. Greece is the only exception, and shows signs of overinvestment.

Underinvestment in advanced economies might hide a provision of public capital by the private sector, something more difficult to argue for developing economies. Notice, however, that privatization of certain services previously supplied by the government not only affects the denominator of $MPKG$; it also impacts its numerator in expression (3) through changes in the public sector share of GDP. This can be, for example, the reason why Gupta et al.´s (2011) estimates show a lower elasticity of public capital in middle income and advanced economies than in LICs. Given tha expression (2) suggests that γ diminishes in a similar proportion than K_g, privatization should not have a substantial impact on our $MPKG$ computations.

Figure 5 contains the temporal evolution from 1990 to 2009 of the mean values of the public and private returns to capital (first row), their standard deviation (second row), and the standard deviation of the ratio of public to private MPK (third row). These results should be interpreted with care given the greater sensitivity to initial conditions as we go back in time. The standard deviation of each of the MPKs reflects efficiency in the distribution of resources across countries, whereas the dispersion of the MPK ratio tells us about differences in the degree of under/overinvestment in public capital. To some extent, this dispersion may reflect the magnitude of divergence in political views towards public participation in investment across countries. Focusing on the first row, the annual means, public and private, have increased since 1990 in the full sample (left chart). The increase is stronger for public capital in all country groups. The rise in the private MPK is, on the other hand, evident in advanced nations (right chart), but not in developing economies (middle panel). The trend in the developed-world private MPK is most likely due to technical change, whereas the divergence between the two MPKs can be due to several factors such as a decrease in the relative inefficiency of the public sector - further research is needed here.

Figure 5: Mean and Dispersion Time Series



The annual standard deviation of the MPK is more closely related to the concept of capital misallocation. In particular, a falling variation suggests more efficient allocation of capital worldwide. The second row in Figure 5 shows that private capital is persistently allocated more efficiently across countries than public capital – a result which may reflect the purer profit-maximizing incentives of private agents, compared with those in government. The figure also suggests that private capital has become more efficiently allocated across nations since 1990, whilst public capital has not. This result is driven by developing countries (middle panel); in this group we see that, in 1990, the standard deviation of the private and public MPK was roughly equal, but there is divergence over the subsequent two decades. The left chart suggests the opposite in advanced economies – the standard deviation of the public MPK falls whilst that of the private MPK does not. Though in 2009, the dispersion of the private MPK remains much smaller than that of the public MPK.

The left panel in the third row shows a decrease in the standard deviation of the relative MPK until about 2003, and a rise after that date. It suggests that governments converged in the degree of underinvestment of infrastructure until 2003 and then diverged. For the developing world (middle panel), there is also divergence in underinvestment policies until 1996.

4 Extensions

In this section we introduce several extensions to our baseline analysis. First, we consider alternatives to our baseline choice of income shares, including an approach in which we are fully agnostic. Second, we review the cross-section results after extending the sample to 74 countries. Third, we set out to correct public capital stocks for inefficiencies in public investment by using a recently developed measure from Dabla-Norris *et al.* (2012). Public investment inefficiencies are notorious in developing and emerging economies, therefore incorporating them into our analysis is potentially important. Fourth, we consider an alternative to the baseline analysis by allowing the price of public and private investment to differ. Finally, we measure the efficiency loss from capital misallocation by employing a calculation of the deadweight loss as in CF.

4.1 Income Shares

Our approach to income shares has so far been to take the overall share of reproducible capital from national accounts data (adjusted for natural capital and the under-estimation of labor income in small firms) and split it using production function regression estimates of the relative output

elasticities of private and public capital. One issue with using this approach is that behind the regression estimates lie strong identification assumptions. A second issue is that we tend not to think of public capital as priced in the same way as other factor inputs (affecting our interpretation of payments to capital in national accounts data). As a means of dealing with these concerns, we take an alternative approach here in which we can be completely agnostic as to actual income shares. The approach is essentially to ask the following question: suppose we don't have any data on income shares, but only data on output to capital ratios (P_yY/P_kK_p and P_yY/P_kK_g). Given the observed pattern in output to capital ratios, what pattern of public/private output elasticities would be needed to rationalize the data if the world is one of perfect capital markets (i.e. with equalized MPKs)? The reader can then think about whether the pattern and magnitudes seem reasonable given whatever prior on output elasticities is held.

Specifically, we assume a counterfactual in which the returns are equalized across countries, and across sectors, opting for $r + \delta = 0.08$ (a depreciation rate of 4% and a hypothetical world real interest rate of 4%). By taking a stance on this hypothetical equalized return to capital we can then back out the output elasticities.[23]

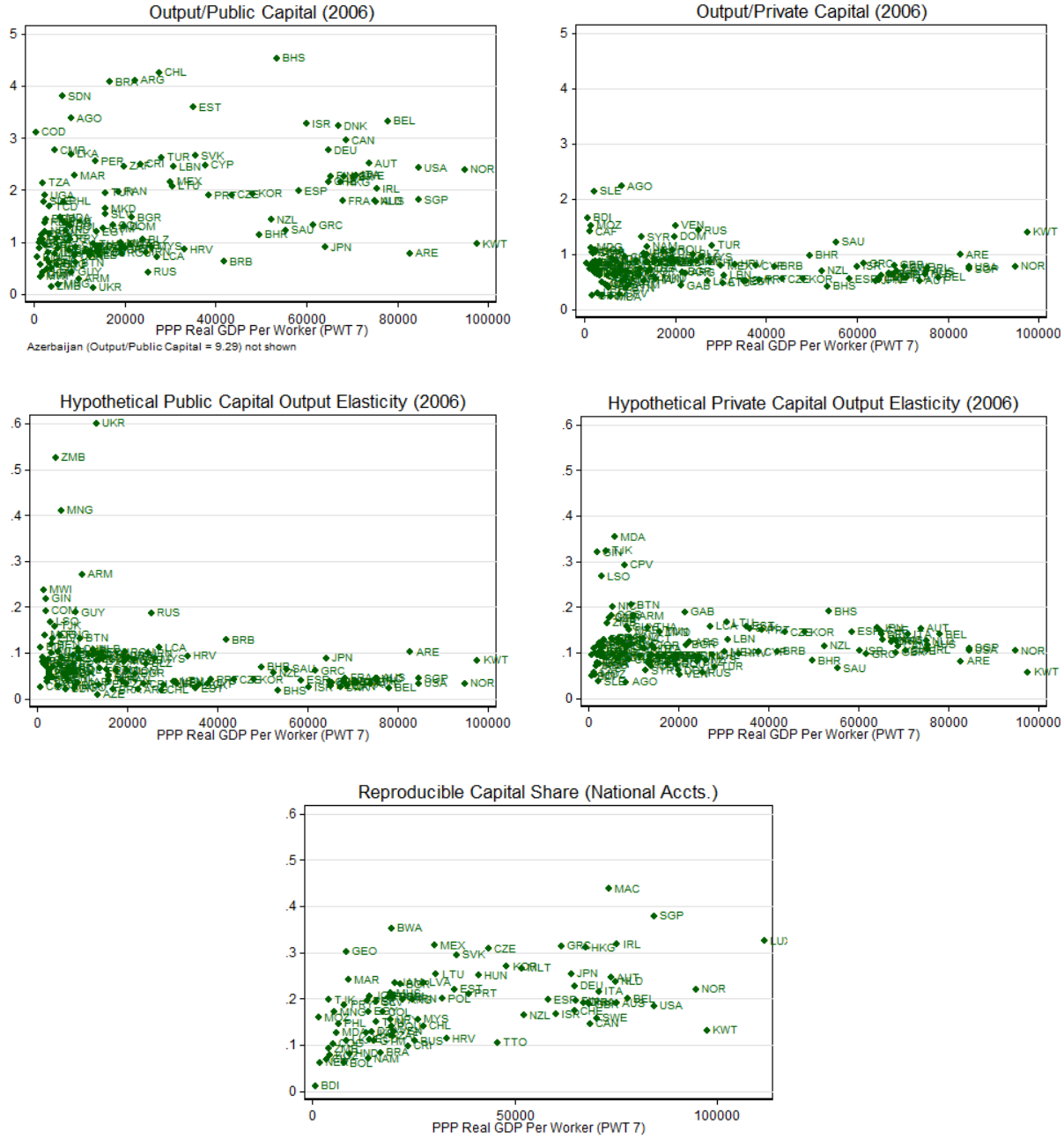
The graphs in Figure 6 use the current price measures of capital and output as in Figure 4 though with a much enlarged sample size given that we no longer need income share data. To ensure the quality of the capital stock data only countries with at least 10 investment share observations and 20 observations of the investment deflator are included. Hypothetical sector-specific output elasticities are calculated simply as $\hat{\beta} = 0.08/(P_yY/P_kK_p)$ and $\hat{\gamma} = 0.08/(P_yY/P_kK_g)$, ensuring that for each country in 2006 we have $MPKP = MPKG = 0.08$.

The first row of Figure 6 is striking. Abstracting completely from income shares, the pattern of output/capital ratios is the familiar one - varied and upward-sloping for public capital, whilst relatively constant and flat for private capital. One possible prior would be that the aggregate production function is common across countries, so that output elasticities are essentially constant in the cross-section. If that is the case, the patterns here hint at the misallocation of public capital alongside the relatively efficient allocation of private capital. These patterns are consistent with profit motives bringing private capital-output ratios in line across countries and political motives keeping the public capital-output ratios out of step.

Row two gives the pattern of counterfactual shares consistent with equalized MPKs. A key

[23] Note that other things equal, a larger choice of $r + \delta$ will lead to larger elasticities, and vice versa.

Figure 6: Counterfactual Shares



aggregate pattern of the public capital share is that, especially for advanced economies, the hypothetical output elasticity is very low (for the US, for example, it is 0.03). For the observed data to be consistent with optimal levels of public capital, the output elasticity of public capital would have to be roughly 0.05 for advanced economies. Whether this is believable depends on our frame of reference. Of course, if we take regression estimates at face value, the implication is that this is not believable, and that rather there is pervasive under-investment in public capital in advanced economies. The hypothetical public capital output elasticity seems more reasonable for developing countries, with some bunching around 0.1 – though there is a great deal of heterogeneity. As for private capital, much of the sample has a hypothetical output elasticity between 0.1 and 0.2, with again more heterogeneity amongst developing countries.

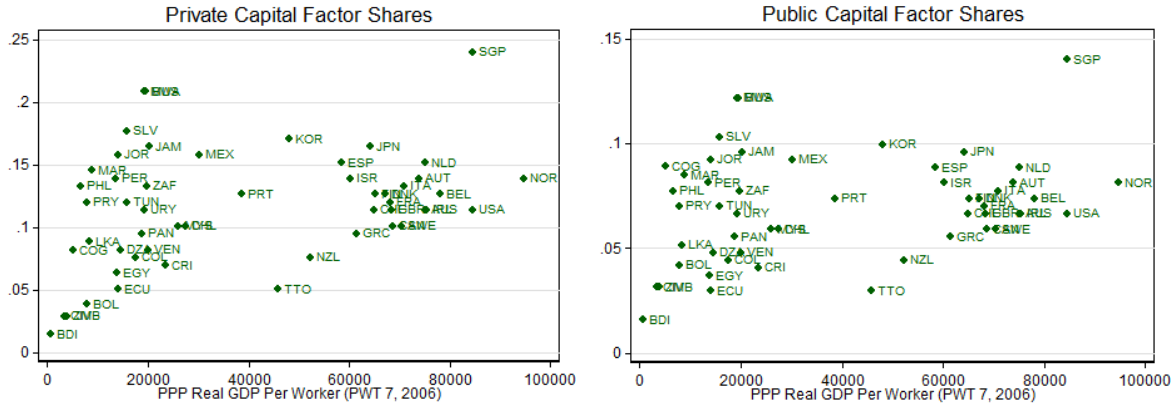
Looking at the bunching in both graphs, a summary observation is that for equalized MPKs we on average require higher public capital output elasticities in developing than advanced economies, and the opposite for private capital. Again, whether this is reasonable depends on one's priors.

An informal way of interpreting these graphs is to ask the question: which pattern of output elasticities is more believable? If we believe the right graph more than the left, we have again the interpretation that private capital is allocated more efficiently across countries than public capital - a central claim of the earlier analysis.

In the final graph of Figure 6 we have again the national accounts income share data using the extended sample of reproducible capital shares of 85 (this extension to be discussed in the next section). Given the motivation for the alternative approach of this section, it may not be helpful here to think of this data as literally the sum of the private and public capital output elasticities. But to the extent that there is some relationship (perhaps most closely with the private capital output elasticity if we assume instead that payments to private factors exhaust output), the positive relationship with income is an additional contradiction of the pattern in the hypothetical output elasticities we find - though notably more a contradiction of the public capital elasticities than the private capital elasticities, given that the bunching of the latter suggest that by and large the private capital elasticity is larger in richer countries. This observation again supports the claim of greater misallocation of public than private capital.

Turning away from agnosticism, we now consider the validity of the income shares actually used in the baseline. The public and private shares for our initial maximum sample of 52 countries are shown in Figure 7.

Figure 7: Factor Shares

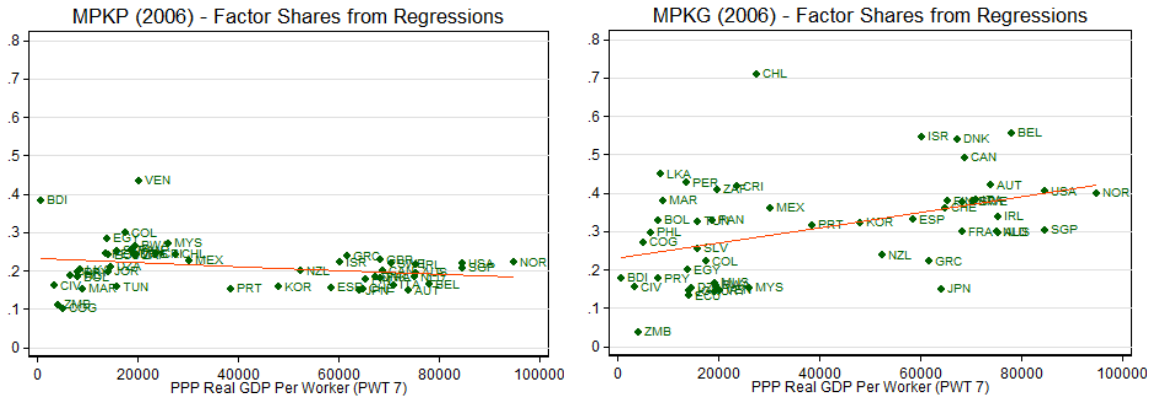


The mean private capital share in income is 0.12, whilst the mean public capital share is 0.07. The mean overall share of reproducible capital is 0.19 – this may seem low given the usual national accounts estimates being around 0.3 or 0.4, but it can be explained by the natural capital adjustment: prior to this adjustment, the mean capital share is 0.35.

One question for these capital share measures is whether they are consistent with other estimates of the output elasticity of capital from the production function regression literature. A useful reference point is given by Bom and Ligthart (2010) who carry out a meta-analysis on 67 studies estimating the private output elasticity of public capital using the production function approach. Even given much variation across the studies, they find the average true output elasticity of public capital to be positive and significant – giving support for the implicit assumption throughout this paper that public capital is productive and should appear in the production function.

More specifically, after correcting for linear publication bias, the unconditional average output elasticity of public capital is found to be 0.146. This is double the mean public capital share in our sample, though it tells us nothing about the relationship of the public capital share with income. Having said that, had we applied our public-private split to the capital share before the adjustment for natural capital, the mean public capital share would be 0.13. Our estimates of the public capital share are then consistent with the production function approach once we take into account the adjustment for natural capital here which is omitted in production function studies. Though many of the studies in Bom and Ligthart's sample are for the US or other advanced economies (and

Figure 8: Applying Regression Estimates



so not completely applicable to the estimates in this paper), the one study which focusses on LICs (Dessus and Herrera (2000)) yields a similar output elasticity of 0.13.

Bom and Ligthart note that the conditional output elasticity of public capital in the benchmark specification, 0.165, implies a public MPK for the US of 28.8-32.6 percent in 2001. Our preferred estimate (using current price local currency data) is 16.9 percent – similarly implying under-investment in public capital by the US, but smaller because of the smaller capital share we calculate.

As another robustness check here, we see how our results change if we apply the regression estimates for β and γ directly, ignoring national accounts estimates for $\beta + \gamma$, and at the loss of full heterogeneity in factor shares.[24] This check allows obtaining the MPKs without imposing the constant returns to scale and perfect competition assumptions, although the estimated regression is derived from a Cobb-Douglas specification. Turning again to columns (2) and (3) in Table 6 of Gupta *et al.*, for LICs we set $\beta = 0.231$ and $\gamma = 0.253$, whilst for middle-income and advanced economies we have $\beta = 0.286$ and $\gamma = 0.167$. The results are shown in Figure 8 for the core sample of fifty countries in 2006.

As is to be expected, this approach yields higher estimates for the MPKs all round – the private MPK looks roughly equalized at around 20 percent (though there are two notable outliers), whilst most public MPKs lie between 10 and 50 percent. These estimates seem implausibly high,

[24] We thank Steve Bond for this suggestion.

showing the advantage of using national accounts data to get at capital shares instead of regression estimates. Still, taking these plots at face value, we find now that the private MPK is slightly downward-sloping (fitted line is negative and significant at 10 percent) whilst once again the public MPK is upward-sloping (fitted line is positive and significant at 1 percent).

4.2 Extending the Sample

The sample of 50 countries in 2006 only includes 5 LICs (Burundi, Bolivia, Côte d'Ivoire, Republic of Congo and Zambia). The major data constraint is the share of reproducible capital in income – it is only available for 52 countries. By largely following Bernanke and Gurkaynak (2001), we successfully extend the sample to 85 countries by re-calculating labor shares using the latest national accounts data from the *United Nations Statistics Division*.[25] With this extended share data it is then straightforward to re-calculate the MPKs, which we do in Figure 9 for 2005.[26] After matching on public and private capital data and imposing one restriction on data quality (requiring a minimum of 10 public investment share observations), the sample is 74 countries, of which 10 are LICs.[27]

The results are largely unchanged in the extended sample, though the private MPK fitted line is now positive and statistically significant at 1 percent. There are also a handful of outliers that emerge – Argentina has a large public MPK for a developing country, whilst Kuwait has a very low public MPK for an advanced economy. In the latter case, it may be Kuwait's oil wealth driving high public investment and low public MPK. Moldova is an outlier in terms of its MPK ratio (excluded from the bottom right graph) – its public MPK is around 8 times its private MPK, a rare feature of a low income country in this dataset.

4.3 Incorporating Inefficiencies in Public Capital Formation

Pritchett (2000) and Caselli (2005) correctly argue that in many countries only a fraction of the actual accounting cost of investment passes into the value of the public capital stock. It is then the case that the public capital stock suffers from an upwards bias measurement problem when the perpetual inventory method is applied to past investment flows. In our case that would imply that the MPK schedules could also be biased – perhaps the public MPK is not upward sloping after all.

[25] See Appendix 2 for full details of the data construction.

[26] We opt for 2005 here rather than 2006 because Niger drops out of the sample in 2006.

[27] The sample size falls primarily because of the following countries lacking investment share data in WEO: Belarus, Hungary, Kyrgyzstan, Luxembourg, Latvia, Macao, Malta and Poland.

Figure 9: Extended Sample

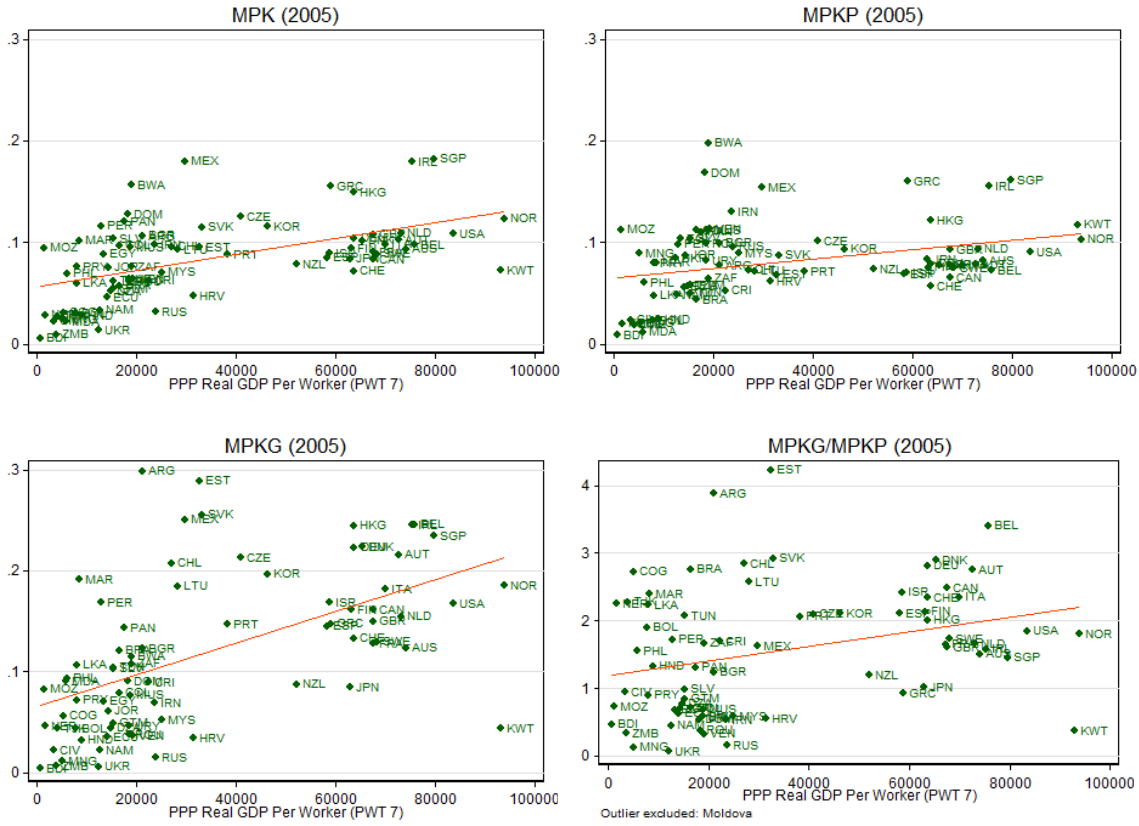


Table 5: PIMI-adjusting and Infrastructure Correlations

Period: 2001-2005	Infrastructure Quantity*	Infrastructure Quality*
Public Capital/GDP	0.1205	−0.0712
PIMI-adjusted Public Capital/GDP	0.5596	0.5573
N	70	47

*Infrastructure Quantity and Infrastructure Quality are from Calderón and Servén (2008).

By overstating the public capital stock in developing countries where public investment efficiency is lower, the public MPK is understated. Here we carry out an efficiency adjustment to the public capital stock measures by using a newly constructed measure, the Public Investment Management Index (PIMI), as our inefficiency proxy.

Using the Dabla-Norris *et al.* (2012) data, correlations in Table 5 suggest that the PIMI-adjusted public capital stock is a better proxy for actual public capital than the unadjusted measure. The unadjusted measure of public capital (as a percent of GDP) is only weakly positively correlated with infrastructure quantity and actually negatively correlated with quality. The PIMI-adjusted measure on the other hand has a fairly strong positive correlation with both. As shown in Dabla-Norris *et al.* (2012), the PIMI possesses a lot of attractive features and is a good first proxy of public inefficiencies; however, it is also only an ordinal measure. Subsequent results using this approach should therefore be interpreted with care.

To obtain the efficiency-adjusted public capital stock ($\bar{K}_{gt}$), we focus on the two components of the PIMI most closely related to investment efficiency – project appraisal and selection.[28] These two components are summed for each country and normalized to lie between zero and one, resulting in a time-invariant efficiency measure ξ. The construction of the public capital stock becomes:

$$P_{kt}\bar{K}_{gt} = (1-\delta_{gt})\left(\frac{P_{kt}}{P_{kt-1}}\right)P_{kt-1}\bar{K}_{gt-1} + \xi I_{gt-1}, \tag{6}$$

so that

$$P_{kt}\bar{K}_{gt} = (1-\delta)^t\left(\frac{P_{kt}}{P_{k0}}\right)\frac{\xi I_{j0}}{\mathfrak{g}_g+\delta_{g0}} + \sum_{i=1}^{t}(1-\delta)^{t-i}\left(\frac{P_{kt}}{P_{k0}}\right)\xi I_{gi-1}, \tag{7}$$

which (because of time-invariance) implies the straightforward adjustment

$$P_{kt}\bar{K}_{gt} = \xi P_{kt}K_{gt}. \tag{8}$$

Furthermore, we assume that efficiency equals one in advanced economies, that is, their public

[28] The other two components are implementation and evaluation.

capital stock is not adjusted.[29] The private share in total capital income is assumed to be 0.66 in MICs and advanced economies; 0.68 in LICs (consistent with Gupta et al. (2011) results using efficiency-adjusted capital).

Figure 10 compares the public MPK measured before ($MPKG$) to the PIMI-adjusted measure ($eMPKG$). The use of PIMI data causes a drop in the sample size from fifty to thirty-seven countries. The result, for both 1996 and 2006, is a potential resolution of the upward-sloping public MPK. However, these results require careful interpretation. Upon introducing the concept of imperfect public investment efficiency, the measure of interest changes. Under the standard assumption of perfect efficiency (i.e. $\xi = 1$ in the capital accumulation equation), the price-corrected MPK reflects well the returns to investment. Once we consider $\xi < 1$, there is a disconnect between investment flows and capital accumulation. Assuming that firms take prices as given, the correct measure of the returns to public investment is now

$$MPIG_t = \frac{P_{yt}}{P_{kt}} \frac{\delta Y_t}{\delta K_t} \frac{\delta \left(P_{kt} K_t\right)}{\delta I_t} = MPKG_t \cdot \xi_t = \eta \frac{P_{yt} Y_t}{P_{kt} \bar{K}_{gt}} \cdot \xi_t, \tag{9}$$

which we call here $MPIG$, the marginal product of public investment. In expression (9), η represents the efficiency-adjusted public capital share.

It is crucial to notice that $\delta(P_{kt}K_t)/\delta I_t$ depends only on efficiency at time t, whereas $\delta Y_t/\delta K_t$ is a function of the full history of efficiency (though with declining weight as we go further into the past). Under the assumptions of *constant* efficiency and same factor shares ($\eta = \gamma$), the $MPIG$ is in fact identical to the un-adjusted $MPKG$ (first row, Figure 10) – this comes as the result of an exact offset with ξ cancelling out. If the factor shares are estimated differently when we take into account public investment efficiency (which is the case here), the $MPIG$ is reflected by the $MPKG$ measured using η rather than γ (third row, first column).[30] Low historic efficiency will imply a smaller capital stock today and a correspondingly higher $MPKG$ (as we see in the second row of Figure 10), but this is offset by the low present-day public investment efficiency. The purpose of all this is to show that the result of an upward-sloping return to public investment across countries is robust to the introduction of inefficiency in public investment, provided the inefficiency is either constant or at least that present-day efficiency is a close proxy for 'historic efficiency'.

[29] There is no PIMI data available for advanced economies – but it is notable that the most developed countries in the PIMI sample tend to have high scores.

[30] The private MPK should also be adjusted accordingly using the efficiency-adjusted factor shares. This is done in the fourth row, second column. There are no qualitative differences to the earlier private MPK schedule.

Figure 10: Adjusted Public MPK

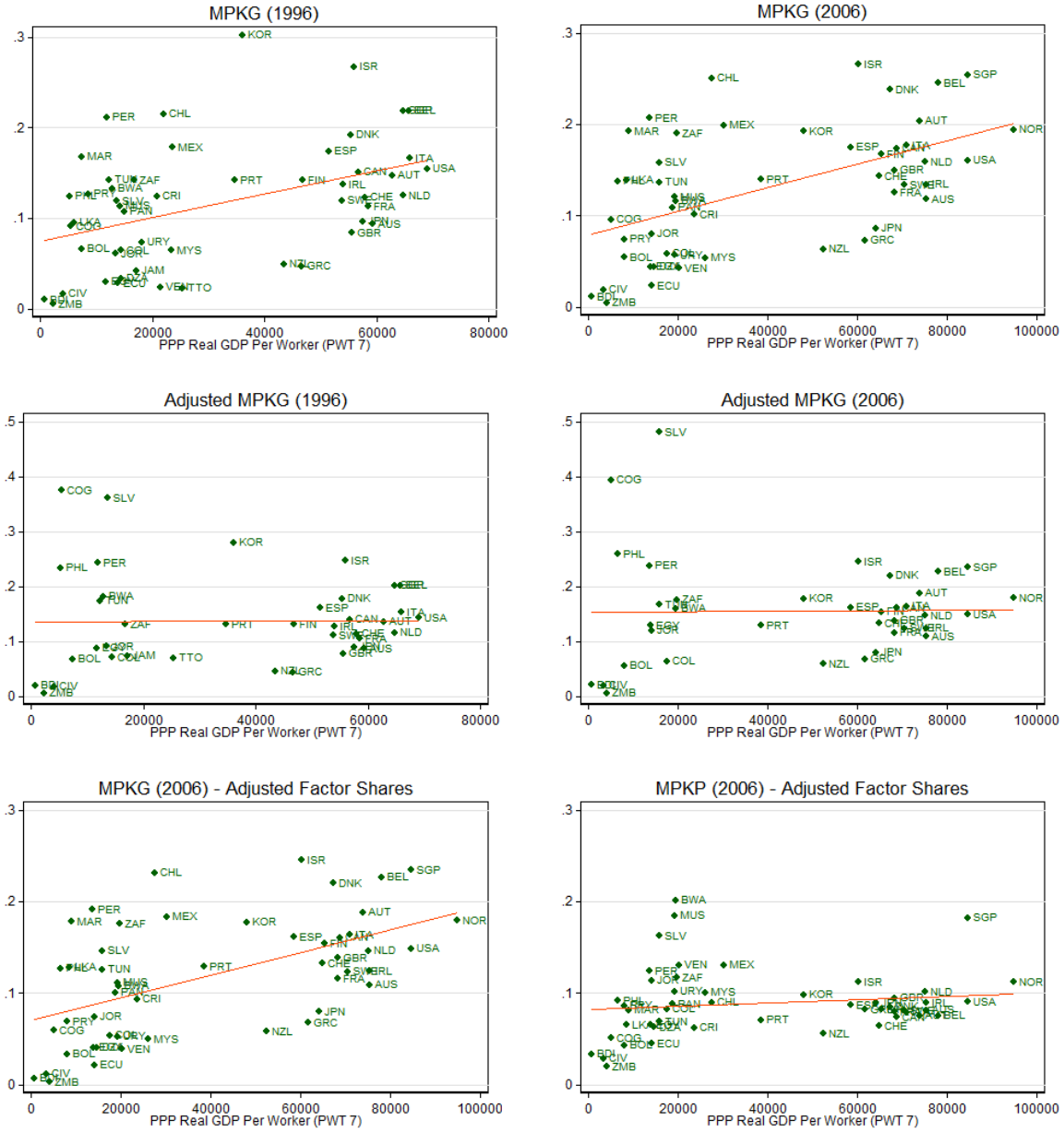
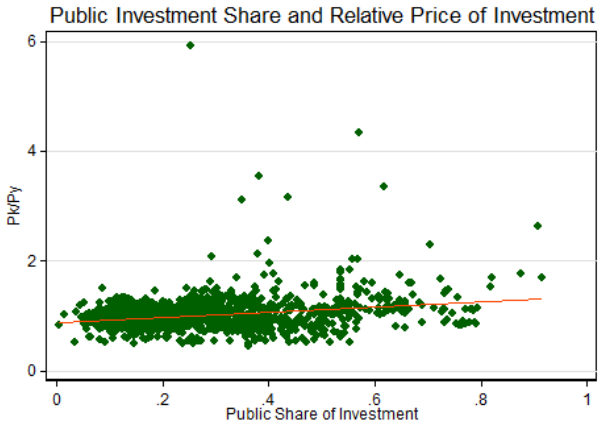


Figure 11: Relative Prices



Even if the $MPKG$ schedule is the most useful for understanding actual public investment returns, the $eMPKG$ schedule may still have a useful interpretation. In some sense it hints at the returns to public investment possible if public investment efficiency in developing countries was brought up to the level of advanced economies, even leaving the greater relative price of capital in developing countries unchanged. From a policy perspective, there seem to be two implications for bringing public returns in developing countries in line with those in advanced economies: reform public investment efficiency, and tackle the higher relative price of capital goods.

4.4 Price Disaggregation

One issue with the baseline analysis is that we implicitly assume (by using a common investment deflator) the price of public and private investment to be equal. This is a sensible first approach since there does not exist investment deflator data disaggregated by sector (public and private). One simple check on this assumption is to plot the public share of investment against the relative price of investment from PWT. A positive association may imply that the public sector pays more for investment goods than the private sector (though this would not be the only explanation). If there is no correlation, the assumption of a common price of investment seems more reasonable. For the sample of fifty, the plot is shown in Figure 11 (where we use data in all time periods):

Clearly there are a handful of outliers, but the mass of points does show a positive association, and the fitted line is positive and significant at 1 percent. The relative price of investment tends

to be higher in countries/years where the public sector share in total investment is higher. This is not conclusive (it can be shown that this association could still be consistent with the price of private investment being higher than that of public investment), but it suggests exploration of the robustness of our results to different assumptions on prices.

One robustness check we can try is to split the price ourselves, using some proxy for the relative price of public to private investment goods. This may affect our baseline results to the extent that, for example, the public sector in developing countries pays more for investment goods relative to the private sector.

A first effort at this is to assume that $P_{k_p}/P_{k_g}$is proxied well by P_C/P_G (the relative price of consumption to government consumption, from PWT 7.0). In principle we would like to know that these two are highly correlated – but it is not obvious how to do this given that P_{k_p}/P_{k_g} is not observed. Still, the intuition would be that the same behavior driving the public/private consumption price differences drives investment price differences.

Recall that our MPK measures are

$$MPKP = \beta \frac{P_y Y}{P_k K_p}; \qquad MPKG = \gamma \frac{P_y Y}{P_k K_g}.$$

Define $a = \frac{I_p}{I}$, the *real* share of private investment in total investment. The adjustment we want to make is $MPKP' = MPKP * \frac{P_k}{P_{k_p}}$ and $MPKG' = MPKG * \frac{P_k}{P_{k_g}}$, where

$$\begin{aligned} \frac{P_k}{P_{k_p}} &= \frac{P_G}{P_C}\frac{P_k}{P_{k_g}} = \frac{P_G}{P_C}\frac{aP_{k_p} + (1-a)P_{k_g}}{P_{k_g}} \\ &= a + (1-a)\frac{P_G}{P_C}. \end{aligned}$$

By similar rearranging,

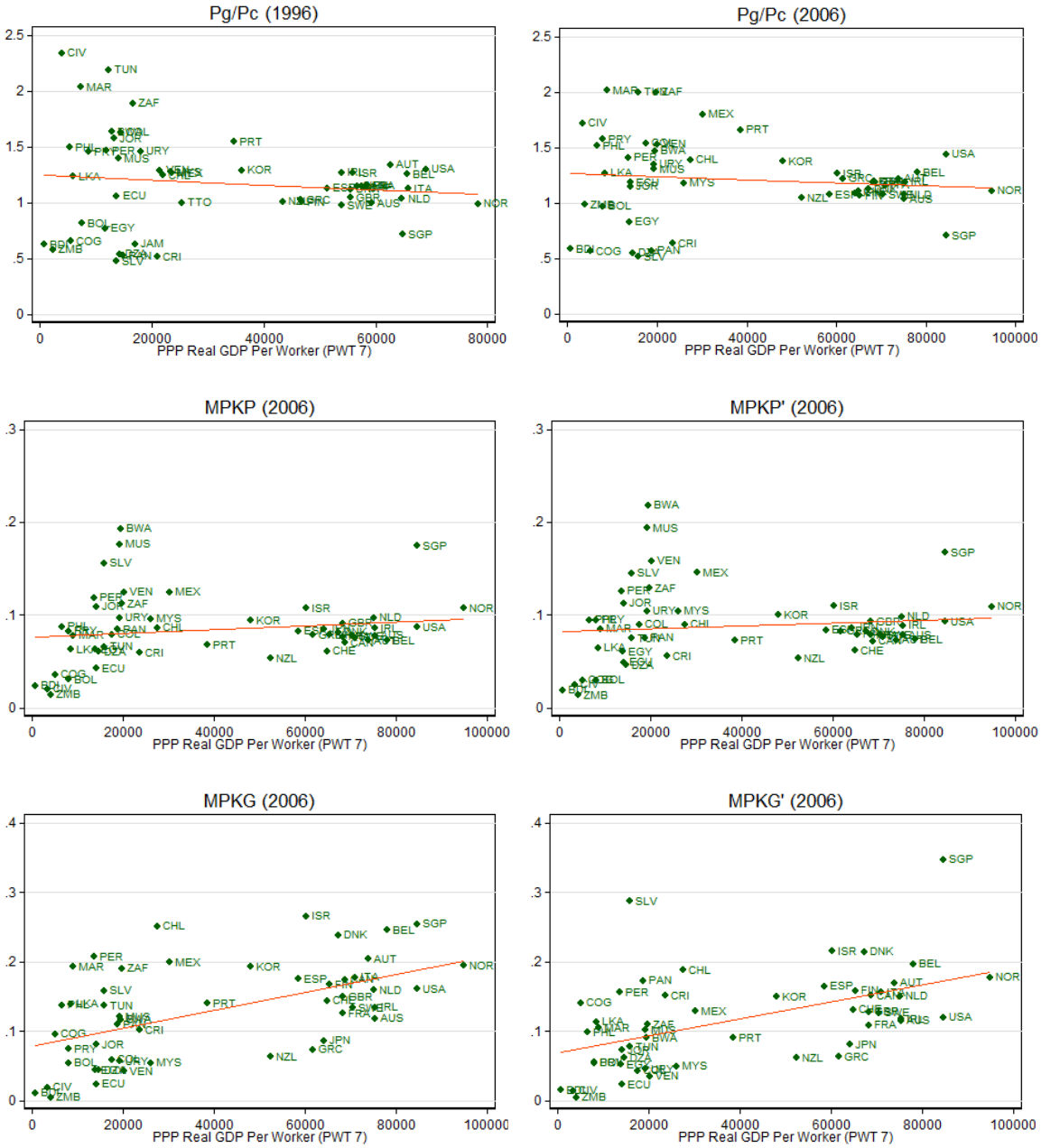
$$\frac{P_k}{P_{kg}} = a\frac{P_C}{P_G} + 1 - a.$$

Therefore, the adjustment to make is

$$\begin{aligned} MPKP' &= MPKP * \left(a + (1-a)\frac{P_G}{P_C}\right), \\ MPKG' &= MPKG * \left(a\frac{P_C}{P_G} + (1-a)\right). \end{aligned}$$

This shows clearly that the adjustment depends on both the private:public consumption price ratio, as well as the private share in investment. Whilst a is not observed, we have WEO data on the

Figure 12: Price-adjusted MPKs



private investment share of overall investment as the best approximation. Figure 12 shows the relative consumption price data and the result of the adjustment in 1996 and 2006.

If anything, P_G/P_C is downward-sloping – suggesting that the prices spent on consumption by governments in poor countries are relatively higher. Applying this to investment prices, the implication is that our results our even stronger – the public MPK is even lower in poor countries because we have so far been understating the price of investment goods by using the common deflator. Having said that, the adjusted MPKs are not much changed visually.[31] The fitted lines are as before – insignificant for $MPKP'$, positive and significant for $MPKG'$.

4.5 Deadweight Loss Calculations

A direct measure for the efficiency loss from capital misallocation is the deadweight loss, which we define here, as in CF, as

$$\frac{\sum (Y_i^* - Y_i)}{\sum Y_i},$$

where Y_i^* is counterfactual GDP with capital (public, private or overall) efficiently allocated. The greatest asset of this measure here is that we can start to quantify the relative losses from public versus private capital misallocation. The calculations extend the approach of CF to account for complementarity of public and private capital in the production function.

We abstract from changes in aggregate capital and assume Cobb-Douglas technology. Each country i has production function,

$$Y_i = Z_i^{\varphi_i} K_{pi}^{\beta_i} K_{gi}^{\gamma_i} (X_i L_i)^{1-\varphi_i-\beta_i-\gamma_i},$$

where Z_i is natural capital and X_i is a measure of labor-augmenting technology. Profit-maximization and price-taking ensure that the following conditions hold for each country i:

$$\begin{aligned} \frac{P_i}{P_{K_p}} \beta_i Z_i^{\varphi_i} K_{pi}^{\beta_i - 1} K_{gi}^{\gamma_i} (X_i L_i)^{1-\varphi_i-\beta_i-\gamma_i} &= MPKP_i, \\ \frac{P_i}{P_{Kg}} \gamma_i Z_i^{\varphi_i} K_{pi}^{\beta_i} K_{gi}^{\gamma_i - 1} (X_i L_i)^{1-\varphi_i-\beta_i-\gamma_i} &= MPKG_i. \end{aligned}$$

[31] Note: the graphs for $MPKG$ and $MPKG'$ now have a different scale, up to 0.4, to include the $MPKG'$ for Singapore.

In the counterfactual case where the returns to private and public capital (separately) are equalized across countries, we have that

$$\begin{aligned}
\frac{P_i}{P_{K_p}}\beta_i Z_i^{\varphi_i}(K_{pi}^*)^{\beta_i-1}K_{gi}^{\gamma_i}(X_iL_i)^{1-\varphi_i-\beta_i-\gamma_i} &= MPKP^*, \\
\frac{P_i}{P_{Kg}}\gamma_i Z_i^{\varphi_i}K_{pi}^{\beta_i}(K_{gi}^*)^{\gamma_i-1}(X_iL_i)^{1-\varphi_i-\beta_i-\gamma_i} &= MPKG^*.
\end{aligned}$$

These conditions can be manipulated to show that the counterfactual capital stocks can be calculated as

$$\begin{aligned}
K_{pi}^* &= \left(\frac{MPKP_i}{MPKP^*}\right)^{\frac{1}{1-\beta_i}} K_{pi}, \\
K_{gi}^* &= \left(\frac{MPKG_i}{MPKG^*}\right)^{\frac{1}{1-\gamma_i}} K_{gi}.
\end{aligned}$$

$MPKP^*$and $MPKG^*$ are however unknown. To solve for these, we require an additional resource constraint – we impose that the aggregate counterfactual private/public capital stock is equal to the existing aggregate stocks:

$$\begin{aligned}
\sum K_{pi}^* &= \sum K_{pi} = \sum \left(\frac{MPKP_i}{MPKP^*}\right)^{\frac{1}{1-\beta_i}} K_{pi}, \\
\sum K_{gi}^* &= \sum K_{gi} = \sum \left(\frac{MPKG_i}{MPKG^*}\right)^{\frac{1}{1-\gamma_i}} K_{gi}.
\end{aligned}$$

We solve for $MPKP^*$ and $MPKG^*$ to an accuracy of four significant figures. Once we know the counterfactual equalized MPKs, it is straightforward to find counterfactual capital stocks country-by-country. Counterfactual income with private capital efficiently allocated is then simply

$$Y_i^* = Y_i \left(\frac{K_{pi}^*}{K_{pi}}\right)^{\beta_i},$$

or with efficient allocation of public capital it is

$$Y_i^* = Y_i \left(\frac{K_{gi}^*}{K_{gi}}\right)^{\gamma_i}.$$

The deadweight loss measure is then calculated as the overall percentage increase in income from capital reallocation. Since the calculations in this section require comparable capital measures

across countries, we calculate real capital measures using PWT 7.0 data, rather than the current price local currency measures used for our preferred measures of the MPK.

Recalling that CF find the deadweight loss to be 0.1 percent of income in 1996 using PWT 6.1 data, we find a comparable result with PWT 6.1 data using our approach to capital stock construction (which differs slightly to CF in its initial conditions and depreciation rates assumed) – we find the deadweight loss to be 0.054 percent of GDP. Using the latest PWT 7.0 data on the same country sample however, we find the deadweight loss for the same year to be 0.31 percent of income – the update to the data itself yields an update to the deadweight loss.

Our interest is more in finding the deadweight loss by sector, for two reasons. Firstly, the figure of 0.1 percent (or 0.31 percent) could understate the actual deadweight loss if public and private capital are complements in the production function – the simplest intuition is that a completely flat overall MPK schedule (deadweight loss of zero) could conceal an upward-sloping public MPK offset by a downward-sloping private MPK (positive deadweight loss in each sector). Secondly, we are interested in quantifying the difference in efficiency losses between the sectors.

The calculations confirm our priors. In 2005, using PWT 7.0 data, the overall deadweight loss is again 0.31 percent. Once we disaggregate capital, we find the deadweight loss in the private sector (assuming the allocation of public capital unchanged) to be only 0.12 percent, whilst the loss from public capital misallocation is 0.54 percent, almost five times greater. In addition, to be most comparable with the idea of an 'overall deadweight loss', we calculate the overall gain to income of first re-allocating private capital efficiently, and then re-allocating public capital efficiently (given the new incomes and $MPKG$s implied by the re-allocation of private capital). The overall gain is 0.67 percent – over twice as much as the estimate found when considering aggregate capital.[32] That said, the economic significance of this loss to global GDP is still relatively minor.

5 Conclusion

Caselli and Feyrer (2007) deliver an intriguing result: after appropriately adjusting the share and relative price of capital, the overall MPK is shown to be broadly the same across a large group of advanced and developing economies, casting doubt on the international capital frictions explanation of the Lucas Paradox. Motivated by the extensively documented and remarkable differences between

[32] This is in fact a lower bound on the deadweight loss since there would be further 'second-order' gains from iterating and continuing to re-allocate private and public capital optimally until both are efficiently allocated though a brief attempt at re-optimisation suggests that these 'second-order' gains are small.

public and private sector incentives, especially in developing countries, we have attempted in this paper to unpack the overall MPK into its public and private components.

First, we have used the most recent data from WDI, WEO and PWT 7.0, and consistent with recent independent work by Francesco Caselli, we have shown that the cross country schedule of the total MPK is not only flat but rather somewhat positively (and significantly) sloped. Second and most important, we have shown that the main driver of the overall MPK schedule is the strongly positively sloped public MPK in developing countries, whilst the private MPK is found to be flat. This surprising finding is subsequently explored by several extensions and a thorough robustness analysis.

In particular, we extended the analysis to incorporate a recently developed index of public investment management inefficiency in our measurement of public capital. While our application of this index is certainly not ideal, modifying public capital for public investment inefficiencies is hugely important as originally argued by Pritchett (2000). Whilst the adjustment suggests a flat public MPK, we show that the measure of interest, the marginal product of investment, is still proxied well by the unadjusted MPK, provided public investment efficiency is roughly constant.

A large number of detailed and thorough robustness tests to the various assumptions made in the construction of public and private MPK have shown that our baseline findings are robust. Whilst we encourage further work on robustness, we have shown clearly that aggregate estimates can provide a very good start in this line of research and that existing aggregate datasets are adequate for taking on the task. Still, such aggregate estimates should be compared against micro evidence which are as crucially important in understanding the pattern of capital flows.

Our findings lend a new set of explanations to the Lucas Paradox, placing public sector idiosyncrasies center stage. The data supports our intuition – public agents act differently to private agents in the context of investment decisions. The result is a vastly different cross-country variation in the marginal product of capital across the two sectors. Our interpretation has emphasized the possibility of overinvestment by governments in developing countries facing few checks and balances and driven by an electoral motive. In contrast, the data points toward *under*investment in advanced economies – perhaps because of greater political pressure for a laissez-faire approach. Still, the interpretation that we suggest should be further explored.

Taking these claims seriously, there are clearly implications for the role of foreign aid in building capital stocks. Importantly, donors should have realistic expectations and reject the notion that

aid (for investment) will have high returns purely because of a chronic lack of capital in low income countries. Our results suggest that returns to public investment are actually lower in LICs than in advanced economies. This is by no means an argument to reduce aid flows, but suggests the need for a shift in emphasis. In particular, there is renewed impetus for 'investing in investing' (to use the terms of Collier, 2010) – channeling aid towards institutional reforms in order to bring down the high relative cost of capital and to raise public investment efficiency. In some sense this is akin to "Big Push" arguments for aid, whilst remaining within the confines of the neoclassical framework.

Beyond this, our approach suggests a refinement of the outlook on aid presented in Caselli and Feyrer (2007). Caselli and Feyrer presented a skeptical view on aid, concluding that greater flows of aid would only be displaced by capital outflows, given the flat MPK. Our disaggregation brings an alternative view. Based on our findings, the provision of foreign aid is not strongly growth enhancing but could rather facilitate inefficient overinvestment by the public sector, since aid is less accountable to returns than private sources of finance. However, given imperfect substitutability between private and public capital in the production function, this overinvestment leads not to capital outflows, but *inflows* of private capital, since the greater stock of public capital raises the returns to private capital.

We started out on this research with one particular prior: that with careful measurement, the financial return to public capital would be found to be relatively high in developing countries reflecting the large needs in education and infrastructure, to name a few. We were stunningly wrong. It may be fitting to close with a story of Tanzania's ability to attract foreign capital. Taking a walk in the busy streets of Dar es Salaam, the capital city, one is impressed by the vibrant private economic activity, entrepreneurship and the many bank branches (local and multinational) scattered across town. One gets the favorable impression that, although at embryonic stages, the private sector operates under close proximity to "market" conditions. A look at public goods (e.g. rail roads and ports) and the provision of public services (e.g. power generation) signals clear deficiencies. Experts correctly insist on the major progress, including in the public sector, that Tanzania has been through over the last two decades as captured by the country's seven percent average GDP growth. But by all accounts this progress is not sustainable unless capital starts to flow inwards from abroad. This paper points to public sector frictions rather than financial frictions or complementarities to low human capital or TFP as the key constraint to enhancing the MPK

and with it, accelerating international capital inflows.

References

[1] Agénor, P-R., (2010) "A Theory of Infrastructure-Led Development," *Journal of Economic Dynamics and Control* 34, 932-950.

[2] Arestoff, F. & C. Hurlin (2010) "Are Public Investment Efficient in Creating Capital Stocks in Developing Countries?" *Economics Bulletin*, 30(4), 3177-3187.

[3] Bai, C., Hsieh, C. & Y. Qian (2006) "The Return to Capital in China," Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, 37(2), 61-102.

[4] Banerjee, A. & E. Duflo (2005) "Growth Theory through the Lens of Development Economics," *Handbook of Economic Growth*, in: Philippe Aghion & Steven Durlauf (ed.), Handbook of Economic Growth, edition 1, volume 1, chapter 7, 473-552, Elsevier.

[5] Baunsgaard, T. & M. Keen (2005) "Tax Revenue and (or?) Trade Liberalization," IMF Working Papers 05/112, International Monetary Fund.

[6] Becker, G. (1957) "The Economics of Discrimination," Chicago: University of Chicago Press.

[7] Bernanke, B. & R. Gurkaynak (2001) "Is Growth Exogenous? Taking Mankiw, Romer, and Weil Seriously," in NBER Macroeconomics Annual, Ben Bernanke and Kenneth S. Rogoff, eds. (Cambridge, MA: MIT Press, 2001), 11-57.

[8] Besley, T. & R. Burgess (2002) "The Political Economy of Government Responsiveness: Theory and Evidence from India," *Quarterly Journal of Economics*, MIT Press, 117(4), 1415-1451.

[9] Bigsten, A., Collier, P., Dercon, S., Fafchamps, M., Gauthier, B., Gunning, J., Isaksson, A., Oduro, A., Oostendorp, R., Pattillo, C., Soderbom, M., Teal, F. & A. Zeufack (2000) "Rates of return on physical and human capital in Africa's manufacturing sector," *Economic Development and Cultural Change*, 48(4), 801-827.

[10] Bom, P. & J. Ligthart (2010) "What have we learned from three decades of research on the productivity of public capital?" working paper, University of Vienna.

[11] Caselli, F. (2005) "Accounting for Cross-Country Income Differences," in *Handbook of Economic Growth*, Philippe Aghion and Steven Durlauf, eds. (Amsterdam: North-Holland Press, 2005), 679-741.

[12] Caselli, F. & J. Feyrer (2007) "The Marginal Product of Capital," *Quarterly Journal of Economics*, 122(2), 535-568.

[13] Chirinko, R. & D. Mallick (2008) "The Marginal Product of Capital: A Persistent International Puzzle," CESifo Working Paper Series 2399, CESifo Group Munich.

[14] Collier, P. (2010) "The Plundered Planet," Oxford University Press.

[15] Dabla-Norris, E., Bramby, J., Keyobe A., Mills, J. & C. Papageorgiou (2012) "Investing in Public Investment: An Index of Public Investment Efficiency," *Journal of Economic Growth*, 17(3), 235-266.

[16] S. Dessus & R. Herrera (2000) "Public Capital and Growth Revisited: A Panel Data Assessment," *Economic Development and Cultural Change*, 48(2), 407-418.

[17] Fama, E. (1980) "Agency Problems and the Theory of the Firm," *Journal of Political Economy*, 88(2), 288-307.

[18] Gollin, D. (2002) "Getting Income Shares Right," *Journal of Political Economy*, 110(2), 458-474.

[19] Gupta, S., Kangur, A., Papageorgiou, C. & A. Wane (2011) "Efficiency-Adjusted Public Capital and Growth," IMF Working Papers 11/217, International Monetary Fund.

[20] Gourinchas, P.-O. & O. Jeanne (2011) "Capital Flows to Developing Countries: The Allocation Puzzle," wokring paper, University of California, Berkeley.

[21] Herrendorf, B. & A. Valentinyi (2008) "Measuring Factor Income Shares at the Sector Level," *Review of Economic Dynamics*, Elsevier for the Society of Economic Dynamics, 11(4), 820-835.

[22] Hsieh, C. & P. Klenow (2003) "Relative Prices and Relative Prosperity," Working Paper No. 9701, National Bureau of Economic Research.

[23] Johnson, S., Larson, W., Papageorgiou, C. & A. Subramanian (2013) "Is Newer Better? Penn World Table Revisions and Their Impact on Growth Estimates", *Journal of Monetary Economics*, 60(2), 255-274.

[24] Keefer, P. & S. Knack (2007) "Boondoggles, Rent-Seeking, and Political Checks and Balances: Public Investment under Unaccountable Governments," *The Review of Economics and Statistics*, MIT Press, 89(3), 566-572.

[25] Knowles, S. (2001) "Are the Penn World Tables data on government consumption and investment being misused?" *Economics Letters*, 71(2), 293-298.

[26] Lucas, R. (1990) "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review*, 80(2), 92-96.

[27] Pritchett, L. (2000) "The Tyranny of Concepts: CUDIE (Cumulated, Depreciated, Investment Effort) Is Not Capital," *Journal of Economic Growth*, 5(4), 361-384.

[28] Robinson, J. & R. Torvik (2005) "White Elephants," *Journal of Public Economics*, 89(2-3), 197-210.

[29] Romer, D. (2006) "Do Firms Maximize? Evidence from Professional Football," *Journal of Political Economy*, University of Chicago Press, 114(2), 340-365.

[30] Taylor, A. (1998) "Argentina and the World Capital Market: Saving, Investment, and International Capital Mobility in the Twentieth Century," *Journal of Development Economics*, 57(1), 147-184.

[31] World Bank (2006) Where is the Wealth of Nations? (Washington, DC: The World Bank, 2006).

6 Appendix 1

In this appendix we present a simple framework that explores the following issues. Why does the public MPK schedule slope up? How does the analysis change if we allow for increasing returns in production? Can other public sector inefficiencies affect our MPK measures? The model is a multisector setup that includes possible externalities from public capital as in Barro (1990).

6.0.1 Production

The economy produces three final goods: a consumption product (Y_c); and two types of investment products – one for the private sector (Y_m) and the other for the public sector (Y_s).[33] The three goods are manufactured by private firms. The government simply finances some of the production using revenues from lump-sum taxes.

Firms in sector j employ private capital (K_{pj}), efficiency-adjusted public capital ($\bar{K}_{gj}$) and other factors (X_j) as inputs. Production benefits from a positive external effect generated by the economy's average stock of efficiency-adjusted public capital; which assuming that firms are a mass of size one equals the total stock $\bar{K}_g$, where $\bar{K}_g = \bar{K}_{gc} + \bar{K}_{gm} + \bar{K}_{gs}$. From the firm's viewpoint, the three goods are manufactured under constant returns to scale and perfect competition according to

$$Y_c = \left(A_c \bar{K}_g^{\phi}\right) K_{pc}^{\beta} \bar{K}_{gc}^{\gamma} X_c^{1-\beta-\gamma}, \tag{10}$$

$$Y_m = \left(A_m \bar{K}_g^{\phi}\right) K_{pm}^{\beta} \bar{K}_{gm}^{\gamma} X_m^{1-\beta-\gamma}, \tag{11}$$

$$Y_s = \left(A_s \bar{K}_g^{\phi}\right) K_{ps}^{\beta} \bar{K}_{gs}^{\gamma} X_s^{1-\beta-\gamma}; \tag{12}$$

Input elasticities are such that $\phi > 0$, $\beta, \gamma \in (0,1)$ and $\beta + \gamma < 1$. Employing results in Herrendorf and Valentinyi (2008), we assume that all sectors display the same input intensities. Total factor productivity (TFP) in private and public investment-goods production are related. In particular, we suppose that $A_s = \varphi A_m$, where $\varphi \in (0,1]$; that is, firms may not be as efficient when they produce for the public sector. The TFP parameters A_c and A_m are assumed to grow exogenously at rate $\mathfrak{g}$.

The idea behind expressions (11) and (12) is that the public and private sectors invest systematically in different types of capital that are complements in the production function. In fact, there is some capital that the public sector is more willing to invest in; historically, for example, the public

[33] Time subcripts are eliminated for simplicity.

sector has been instrumental in the provision of transportation networks or sanitation in many nations. It is also interesting to note that the above production functions have the returns-to-scale structures in Barro (1990) and in CF as special cases. In particular, when γ equals *zero*, functions (10) to (12) look like in Barro (1990): there are constant returns over private inputs, and public capital boots TFP. If, on the other hand, the parameter equalized to *zero* is ϕ, there are constant returns to scale over all inputs as in CF.

As mentioned above, production can be financed either by private agents to increase firms' stocks or by the public sector to provide infrastructure to the economy. The motion equations for capital are given by:

$$P_k\dot{K}_p = I_p - \delta P_k K_p, \text{ with } I_p = P_k Y_m \tag{13}$$

$$P_s\dot{\bar{K}}_g = \xi I_g - \delta P_s \bar{K}_g, \text{ with } \xi I_g = P_s Y_s, \tag{14}$$

where $K_p = K_{pc} + K_{pm} + K_{ps}$. Expressions (13) and (14) are the continuous-time version of motions (4) and (6), taking into account that the price of public capital (P_s) can differ from the one of private capital (P_k).

As in the previous section, the coefficient ξ is a measure of government inefficiency in channeling funds to investment in line with Agénor (2010). Following the same steps as in the derivation of expression (8), it is straightforward that (14) implies that the public-capital stock corrected for inefficiencies ($\bar{K}_g$) and the one not corrected (K_g) maintain the relationship:

$$P_s\bar{K}_g = \xi P_k K_g. \tag{15}$$

Let us consider that firms pay an interest rate r to private savers for the capital borrowed ($P_k K_{pj}$) to construct their capital, a price τ (net of depreciation) for the use of public infrastructure, like airport fees and taxes, and a rate w to each unit of other inputs hired. An alternative environment for the public input would be to think that the use of public infrastructure is given by the amount supplied by the government in the area where the firm is located. In this case, τ would be the endogenously determined shadow price of the public input.

Profit maximization by individual firms in production sector $j \in \{c, m, s\}$ implies the following FOCs for capital and other inputs:

$$r = \left(\frac{P_j}{P_k}\right)\frac{\partial Y_j}{\partial K_{pj}} - \delta, \tag{16}$$

$$\tau = \left(\frac{P_j}{P_s}\right)\frac{\partial Y_j}{\partial \bar{K}_{gj}} - \delta, \tag{17}$$

and

$$w = P_j \frac{\partial Y_j}{\partial X_{gj}}, \tag{18}$$

where P_c represents the price of the consumption good.

It is easy to show that expressions (16) to (18) imply that capital-labor ratios need to be equalized across sectors, that the relative prices of goods are exclusively pinned down by the relative TFPs, and that the private-to-public capital ratio depends on input elasticities and prices. Mathematically,

$$\frac{K_{pc}}{L_c} = \frac{K_{pm}}{L_m} = \frac{K_{ps}}{L_s} = \frac{K_p}{L}, \tag{19}$$

$$\frac{\bar{K}_{gc}}{L_c} = \frac{\bar{K}_{gm}}{L_m} = \frac{\bar{K}_{gs}}{L_s} = \frac{\bar{K}_g}{L}, \tag{20}$$

$$\frac{P_c}{P_k} = \frac{A_m}{A_c}, \tag{21}$$

$$\frac{P_s}{P_k} = \frac{1}{\varphi}, \tag{22}$$

and

$$\frac{K_{pc}}{\bar{K}_{gc}} = \frac{K_{pm}}{\bar{K}_{gm}} = \frac{K_{ps}}{\bar{K}_{gs}} = \frac{\beta}{\gamma}\frac{\tau + \delta}{r + \delta}, \tag{23}$$

where

$$L_c + L_m + L_s = L, \tag{24}$$

$$K_{pc} + K_{pm} + K_{ps} = K_p, \text{ and } \bar{K}_{gc} + \bar{K}_{gm} + \bar{K}_{gs} = \bar{K}_g. \tag{25}$$

The total amount of other resources X is assumed to grow at rate n.

Expression (22) implies that the relative price of public infrastructure is affected by inefficiencies related to the relative TFP. Taking this into account, we can use equality (15) to get

$$\bar{K}_g = \xi\varphi K_g. \tag{26}$$

Inefficiencies reduce the amount of resources that end up being public capital in real terms.

6.0.2 Efficiency Adjusted MPKs

The MPK measures that we used to proxy the return to private and public capital investment are contained in expression (3). In terms of our model, GDP in the numerator of the last expression equals

$$P_y Y = P_c Y_c + P_k Y_m + P_s Y_s. \tag{27}$$

The issue that we explore in this section is whether these MPK measures continue being a good proxy for the return to investment once inefficiencies are taken into account.

Given that neither ξ nor φ affect the private sector return, the measure $MPKP$ still captures the financial return to private investment. For the public sector, however, the $MPKG$ no longer gives the relevant financial return to investment. As argued above, this return now equals the efficiency-adjusted marginal product of investment, expression (9); although we need to substitute P_s for P_k in (9) since the two prices may differ because of φ. The proxy $MPIG$ becomes:

$$MPIG = \frac{P_y}{P_s}\frac{\partial Y}{\partial \bar{K}_g}\frac{\partial\left(P_s\bar{K}_g\right)}{\partial I_g}.$$

Which using expressions (14), (22) and (26) can be written as

$$MPIG = \frac{P_y}{P_s}\frac{\partial Y}{\partial \bar{K}_g}\xi = \frac{P_y}{\frac{P_k}{\varphi}}\left(\gamma\frac{Y}{\xi\varphi K_g}\right)\xi = MPKG. \tag{28}$$

Therefore, neither ξ nor φ, as long as they are constant, matter for the calculation.

We already explained in the previous section the intuition for the absence of impact of ξ. For the TFP inefficiency φ, the intuition is similar: the parameter φ affects the stock of public capital, and also its price; both affect the value of the MPK in opposite directions, offsetting each other. Also common to both inefficiencies is that their effect on the capital stock is a function of the full history of efficiency, whereas their other (offsetting) effect depends only on efficiency at time t. Nevertheless, there is an important difference between the two: unlike for the investment inefficiency ξ, the neutrality finding for the TFP inefficiency φ depends on the Cobb-Douglas form adopted by the model.

6.0.3 Socially Optimal MPKs

There exists the possibility that public capital raises TFP across sectors, introducing increases returns. The analysis so far has shown the returns to capital from the firm's point of view. However, a social planner realizes that the return to public capital is higher, and equal to

$$MPKG^{soc} = (\phi + \gamma)\frac{P_y Y}{P_k K_g}.$$

Therefore, if public capital externalities do exist, our measure of the marginal product of public capital will undervalue it.

Given that the analysis is performed in a decentralized economy, the model predictions obtained above still hold. In this scenario, the government tries to achieve the first best by subsidizing the

use of public goods (i.e., τ), or by supplying the socially optimal amount of infrastructure if the use of public goods is free of charge.

On the measurement side, one way to deal with this potential problem of increasing returns is to use regression estimates of the elasticities, instead of input shares from national accounts. This exercise was carried out in Section 4. There, we also took into account that when shares depend on the amount of capital, as it is the case in production function estimation, these shares would be different depending on whether or not capital stocks were efficiency corrected. Importantly, results from these exercises are in line with the baseline.

6.0.4 The Different Behavior of MPKP and MPKG

Next, we look at the price-corrected MPK proxy constructed by CF ($PMPKL$) using the total capital stock. It is easy to show that this measure is simply a weighted average of the public and private MPKs. In particular,

$$PMPKL = \frac{P_j}{P_k}\left(\frac{K_p}{K}\frac{\partial Y_j}{\partial K_{pj}} + \frac{K_g}{K}\frac{\partial Y_j}{\partial K_{gj}}\right), \tag{29}$$

with $K = K_p + K_g$. The equality must hold for all j. Focusing on the consumption-goods production activity, we can expand (29) using (19), (24) and (25) to obtain

$$PMPKL = \frac{K_p}{K}\beta\frac{P_cY_c}{\frac{L_c}{L}P_kK_p} + \frac{K_g}{K}\gamma\frac{P_cY_c}{\frac{L_c}{L}P_kK_g} = (\beta+\gamma)\frac{P_cY_c}{P_kK\frac{L_c}{L}}.$$

Which employing the value of consumption-goods production implicit in (32) (see below) delivers

$$PMPKL = (\beta+\gamma)\frac{P_yY}{P_kK}. \tag{30}$$

It provides a direct measure of the MPK that, under the constant returns to scale assumption, can be obtained using the physical capital share in income to approach $\beta+\gamma$, GDP, and the non-adjusted capital stock. The key prediction of the multisector framework that CF exploit is that the financially-relevant MPK depends on the relative price of final-to-capital goods, P_y/P_k.

Equality (30) is, however, a good proxy only if there are not capital externalities and resources are efficiently allocated between capital types. To see this, we need to compare $PMPKL$ to the financial return to private investment ($MPKP$), which in our model is the right measure of the market return. Focusing again on the c sector,

$$MPKP_c = \beta\frac{P_cY_c}{P_kK_{pc}}. \tag{31}$$

Employing expressions (10), (11), (19) to (22), (24) and (25), we can rewrite (27) as

$$P_y Y = P_c Y_c \, \frac{L}{L_c}. \tag{32}$$

This and (23) make (25) become

$$MPKP = \beta \frac{P_y Y}{P_k K_p} = \left(\beta + \gamma \frac{r+\delta}{\tau+\delta} \right) \frac{P_y Y}{P_k K}. \tag{33}$$

Comparing $PMPKL$ and $MPKP$, they differ because input prices play a role in the latter measure. In fact, when payments to both types of capital are the same, that is, the public sector charges a fee for the use of public infrastructure equal to the market return ($\tau = r$), $MPKP$ equals $MPKG$, and $PMPKL$ and $MPKP$ coincide.

The two prices can, however, differ at least for three reasons. First, government's inefficiencies. Their effect is that the public sector needs to borrow more than private firms to obtain the same amount of physical capital; and therefore, pay a larger price for each unit borrowed. Second, the government may think that the return to public investment differs from the one of private investment – i.e., that $\phi > 0$ – and want to subsidize the use of K_g. A third possibility is that the subsidization is a consequence of political reasons; for example, if the public sector wants to signal its capacity to increase people's welfare.

Our last task is trying to explain why the public marginal product of capital slopes up with income per capita in the cross-section of nations. According to the model, expressions (3), (22) and (23) imply that the ratio public-to-private MPK is given by

$$\frac{MPKG}{MPKP} = \frac{\gamma \frac{P_y Y}{P_s K_g}}{\beta \frac{P_y Y}{P_k K_p}} = \frac{\gamma}{\beta} \frac{P_k}{P_s} \frac{K_p}{K_g} = \frac{\gamma}{\beta} \varphi \frac{\beta (\tau+\delta)}{\varphi \gamma (r+\delta)};$$

that is,

$$\frac{MPKG}{MPKP} = \frac{\tau+\delta}{r+\delta}. \tag{34}$$

Equality (34), suggests that to explain why the public-to-private MPK ratio slopes up, we need to explain why the user cost (or shadow price) of public infrastructure increases with development. One possible explanation suggested by the production functions could be that the external effect from public capital becomes weaker (ϕ decreases) as economies develop, similar to the existence of *big push* effects. In any case, this remains an open question.

7 Appendix 2

This appendix provides details on how we extend the baseline Caselli-Feyrer sample from 52 to 85 countries. We first, calculate Bernanke and Gurkaynak's "Actual OSPUE" (Operating Surplus of Private Unincorporated Enterprises) measure as:

$$Lab.share = \frac{Corporate\ employee\ compensation}{GDP - Net\ indirect\ taxes - Mixed\ income}, \tag{A1}$$

where Mixed Income is the closest variable to OSPUE in the latest (1993) System of National Accounts. This definition of the labor share works on the assumption that the share of labor income in Mixed Income is the same as its share in the corporate sector. Since Mixed Income data is not always available, we calculate "Imputed OSPUE" as above, but with Mixed Income imputed as:

$$Mixed\ income = (1 - \frac{Employees}{Workforce})(Gross\ OS + Corporate\ employee\ compensation), \tag{A2}$$

where OS is Operating Surplus. As Bernanke and Gurkaynak explain, the assumption is made here that the corporate share of overall private-sector income is the same as the share of the labor force employed in the corporate sector. Finally, we calculate the labor force correction (LF) by scaling up aggregate labor income by the ratio of total labor force to the number of corporate employees:

$$Lab.share = \frac{Corporate\ employee\ compensation}{Corporate\ share\ of\ LF * (GDP - Net\ indirect\ taxes)}, \tag{A3}$$

assuming that corporate and non-corporate workers earn the same average wage. Whereas Bernanke and Gurkaynak yield a time-invariant country-level labor share by averaging each measure across 1980-1995, we average across 1990-2010. The sample size is significantly larger with the updated data. When combining the three measures (Actual OSPUE if available, then Imputed, then LF) Bernanke and Gurkaynak have a sample of 53 countries, which falls by one after making the natural capital adjustment. With our new data, the sample size is 83 countries, falling to 70 after the natural capital adjustment. This said, the new sample actually lacks a number of countries (15) that are present in the original sample. Our approach then is to combine the Bernanke and Gurkaynak data for 1980-1995 with our updated data for 1990-2010 in order to maximize the sample size whilst using the best quality data where available.

A further caveat remains – on closer inspection, the Mixed Income data in the latest national accounts appears to regularly differ significantly from OSPUE data used in previous studies. This is

clear when looking at particular country-years for which both variables are known. The tendency seems to be for Mixed Income to be lower than OSPUE, causing our Actual OSPUE measures to be on average lower than BG, and capital shares correspondingly larger. As one example, for the UK we find an Actual OSPUE measure of 0.66 where Bernanke and Gurkaynak found 0.75. No such significant differences exist for the Imputed OSPUE and LF measures for the UK which are 0.74/0.75 in our data and 0.72/0.74 in Bernanke and Gurkaynak's. Our interpretation is that the updated Actual OSPUE data is suspect, and tends to understate labor's share. In combining the available data we then do the following: we use Bernanke and Gurkaynak's Actual OSPUE if available, and if not, we use our updated Imputed OSPUE, then updated LF, then Bernanke and Gurkaynak Imputed, then Bernanke and Gurkaynak's LF, then finally updated Actual OSPUE. This results in a set of labor shares for 98 countries, of which 18 of the observations are our updated Actual OSPUE. The share of overall capital is derived by subtracting each labor share from one.

A final adjustment for natural capital is needed to get to measures of the share of reproducible capital in income. For this we simply follow Caselli and Feyrer (2007) but use updated World Bank wealth estimates from 2005 where available, or the previous data from 2000 if not. After this adjustment, we have α_k for 85 countries.