Matching up the data on education with economic growth models*

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Abstract

The growth literature has not yet fully established how data on educational attainment should be introduced in theories involving human capital. This paper examines alternative specifications of human capital within the Mincerian class that once incorporated in standard growth models may generate predictions that match up with the existing data on education. First, we analyze predictions in the Bils and Klenow (2000) model and show that although they behave correctly under some parameterizations, they are also subject to some shortcomings. Next, we present a standard neoclassical two-sector growth model that adopts a human capital specification in which the fraction of individual's time endowment in school is viewed as an investment rate. We show that the optimally chosen educational attainment predicted by the calibrated model does not correspond to the data as it is unrealistically high. Finally, we propose an alternative specification of human capital based on a law of motion of educational attainment that successfully matches up with the data.

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> The role of human capital needs clarification, in both theoretical and empirical terms. Human capital is widely agreed to be an important factor in economic growth. Maybe more to the point it seems to offer a way to reconcile the apparent facts of convergence with the model. One difficulty is that the measurement of human capital is insecure. School enrollment data are fairly widely available, but they clearly represent a flow, not a stock. Direct measurement of the stock runs into deep uncertainty about depreciation and obsolescence and about equivalence of schooling and investment in human capital. Mention has already been made of the use of relative wages as indicators of relative human capital; the well-known Mincer regressions can also be used, as in Klenow and Rodgríguez-Clare (1997b).
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> Robert M. Solow (*Handbook of Macroeconomics*, 1999 p. 663)

1 Introduction

Even though human capital has been established as one of the primary engines of economic growth and development, it has not yet been fully established how data on educational attainment should be introduced in growth models. It is widely accepted that combining educational attainment data (i.e., Barro and Lee, 1993, 2000) with labor income data obtains the most reliable approximation of human capital across countries. Educational attainment data capture formal education, whereas labor income data are meant to capture the remaining components of human capital such as informal education, i.e., on-the-job training and work experience, (see Stokey, 1988) and health (see Schultz, 1989), for which data are either unavailable or unreliable. In a recent contribution, Mulligan and Sala-i-Martin (2000 p. 216) indicate that employing only labor income data to construct human capital stocks is rather problematic, and that the use of educational attainment data is essential.[1]

An important step in this direction is Bils and Klenow (2000) (BK hereafter). They suggest incorporating human capital into the aggregate production function following a Mincerian approach. In particular, BK specify a human capital production function that is based on the "Macro-Mincer" wage equation, inspired by the labor economics literature, and empirically tested by Heckman and Klenow (1997), Krueger and Lindahl (2001), and Temple (2001), among others. This interpretation has been adopted by many recent papers including Jones (1997, 2002) and Hall and Jones (1999), just to name a few.

In this paper, we follow BK and ask the question: In the context of our theories of economic growth, what is an appropriate specification of human capital within the Mincerian class to which a measure of workers' average years of schooling successfully corresponds? Our primary goal is to search for specifications that, in a simple calibrated growth model, can successfully produce

[1] See Wößmann (2003) for a recent survey on this topic.

educational attainment levels comparable to those observed in the data.[2]

We start by studying steady-state predictions regarding educational attainment in the finite-horizon model presented in BK. BK show that their human capital specification is capable of producing empirically-supported values of educational attainment, namely, values between 8 and 14 years. Our investigation of the robustness of this finding suggests that, even though the BK setup generates reasonable predictions for some parameterizations, the specification presents two problems. First, their choice of rich human capital technology makes it difficult to obtain endogenously values for all prices, either analytically or numerically. This may be the reason why these authors resort to the small-open economy assumption and use exogenous interest rates. Second, as agents' time horizon increases and we get closer to an infinite horizon framework, the predicted levels of educational attainment by the BK model become very sensitive, and the parameter set that delivers empirically supported predictions shrinks considerably.. The latter caveat is especially important because the infinite horizon framework is the workhorses in modern macroeconomic theory.

In the rest of the paper, we search for human capital specifications that could be more appropriate in the infinite-horizon framework. In a first attempt we consider a human capital specification proposed in recent papers, Bils and Klenow (1996) and Jones (1997, 2002), among others, in a very basic neoclassical two-sector growth model. In order to reconcile agents' finite schooling levels with the infinite horizon model, these papers propose interpreting average years of schooling as the fraction of an individual's time endowment allocated to accumulating skill; that is, they interpret average years of schooling as an investment rate.[3] By using the proposed interpretation of educational attainment this problem is potentially resolved. In addition, this interpretation is shown to be particularly useful in empirical tests of the relationship between income growth and human capital (see Jones, 2003). Our main finding is that the calibrated model implies an optimally chosen educational attainment level that has a lower bound of more than 21 years, clearly a much higher value than that implied by the data. We interpret this finding as suggesting that the human capital technology used in this model is too simple to produce realistic educational attainment levels.

Finally, we present a growth model that employs a second alternative specification of human

[2] As Klenow and Rodríguez-Clare (1997) suggests, being able to generate predictions that agree with the data is a desirable property for growth models.

[3] More precisely, these papers attempt to take issue with a potential shortcoming inherent in the infinite horizon neoclassical growth model. That is when individuals face an infinite horizon they invest in education every period, which can make the educational attainment grow with the economy, which is counter-factual.

capital. The proposed specification provides a map between schooling investment and human capital accumulation through a law of motion of educational attainment. A calibration exercise reveals that the model is successful in predicting educational attainment levels consistent with those observed in the data.

The rest of the paper is organized as follows. Section 2 investigates the steady-steady properties of the BK model. Section 3 studies predictions using the specification used in recent papers considering years of schooling as an investment rate (a flow variable). Section 4 proposes an alternative specification of human capital that features a law of motion of educational attainment. Section 5 concludes and provides directions for future research.

2 The Bils-Klenow (BK) Model

Consider an economy that contains households and firms. There is a continuum of finitely-lived generations of agents in the economy. The size of the cohorts grows at rate n. Each agent lives T periods and is endowed with one unit of labor in each of them. The agent allocates this unit to schooling from age 0 to age s, and to consumption-good production from ages s to T.

Firms in the economy produce a unique product, a consumption good (Y), from the combination of efficiency units of labor and the stock of physical capital (K) according to the following aggregate production function:

$$Y(t) = K(t)^{1-\alpha}\left[A(t)\,H(t)\right]^{\alpha}, \quad 0 < \alpha < 1\,; \tag{1}$$

where A is the level of labor-augmenting technology that grows exogenously at rate g_A; H is the skilled-labor input; and α is the share of technology-augmented skilled labor.

The variable H can be also called the aggregate level of human capital in the economy, and is given by

$$H(t) = \int_s^T h(a,t)L(a,t)\;da; \tag{2}$$

where $h(a,t)$ and $L(a,t)$ are human capital and number of workers, respectively, in cohort a at date t. Under this technology, different units of human capital are perfectly substitutable.

Individual human capital stock is characterized by the following equation:

$$h(a,t) = h(a+z)^{\phi}e^{f(s)+g(a-s)}, \;\; \forall a > s. \tag{3}$$

A value of ϕ greater than zero captures a positive external effect from the teacher's human capital into the student's h. Teachers are supposed to be z generations older than students. The exponential term follows the Mincerian interpretation of human capital. The function $f(s)$ simply states that productivity from education depends on the number of periods spent in school, s. The worker's productivity is also affected by work experience, and this is captured by the function $g(a-s)$. Notice that the term $(a-s)$ represents the number of periods that an agent of age a has already spent at the workplace. The derivatives $f'(s)$ and $g'(a-s)$ correspond to the return to schooling and experience estimated in a Mincerian wage regression, respectively. More specifically, $f'(s)$ represents the percentage increase in a worker's wage rate (w) if he accumulates an additional period of schooling, and $g'(a-s)$ provides the same information but related to an additional period of experience.

Consumers' problem is given by

$$\max_{\{s,\ c(t)\}} \int_0^T e^{-\rho t}\left[\frac{c(t)^{1-1/\sigma}}{1-1/\sigma}\right] dt + \int_0^s e^{-\rho t}\xi\ dt, \tag{4}$$

subject to,

$$\int_s^T e^{-\int_0^t r(v)dv} w(t)h(t)\ dt \geq \int_0^T e^{-\int_0^t r(v)dv} c(t)\ dt + \int_0^s e^{-\int_0^t r(v)dv} \mu w(t)h(t)\ dt; \tag{5}$$

where c is consumption; ρ is the discount rate; ξ is flow utility from going to school (see, e.g. Schultz (1963)); r is the interest rate; and μ is tuition costs as a fraction of the opportunity cost of going to school.

From this optimization problem, the First Order Condition (FOC) with respect to s gives that at steady state

$$(1+\mu)w(s^*)h(s^*) = \xi c(s^*)^{1/\sigma} + \int_{s^*}^T e^{-r^*(t-s^*)}\left[f'(s^*) - g'(t-s^*)\right] w(t)h(t)\ dt;$$

where $(*)$ denotes steady-state values. In addition, assuming that $\phi = 0$, and adopting the following specific functional forms:

$$f(s) = \frac{\theta s^{1-\psi}}{1-\psi}, \tag{6}$$

and

$$g(a-s) = \gamma_1(a-s) + \gamma_2(a-s)^2, \tag{7}$$

reduces the FOC to

$$(1+\mu) = \xi\frac{c(s^*)^{1/\sigma}}{w(s^*)h(s^*)} + \int_s^T e^{[g_A+\gamma_1+\gamma_2(a-s^*)-r^*](a-s^*)}\left[\theta s^{*-\psi} - \gamma_1 - 2\gamma_2(a-s^*)\right]\ da. \tag{8}$$

Table 1: Benchmark parameter values

θ	0.323	μ	0.5	ξ	0
ψ	0.58	T	54.5	g_A	0.02
γ_1	0.0512	γ_2	-0.00071	σ	1

The parameter g_A is present in the right-hand-sight of equation (8) because along the balanced-growth path the wage $w(t)$ grows at rate g_A. Expression (8) is identical to equation (16) in BK, and determines the optimal educational attainment for the consumer.

2.1 Calibration and predictions of the BK model

We are now ready to generate predictions about the optimal s^*. Let us take as benchmark parameter values those used in BK. These values are presented in Table 1. Note that using $\xi = 0$ reduces the marginal benefit from education, and implies lower predicted values for s^*. This has the same effect as that generated by the elimination of the externality in the schooling sector ($\phi = 0$). However, given that we want to show that s^* gets implausibly large as T rises, the assumption $\xi = \phi = 0$ has no impact on our main conclusion.

In order to make predictions, we still need to assign values to the interest rate. In a closed economy version of the BK model, solving for r^* either analytically or numerically is difficult. BK then considers a small-open-economy scenario under which r is assumed exogenous to the domestic economy. This assumption comes without considerable loss of generality because expression (8) does not depend on the main determinants of the interest rate: the discount rate ρ, and the elasticity of consumption substitution σ. So, in principle, it should be easy to experiment with estimated values of ρ and σ to generate the desired empirically supported r^*.

We obtain a value of r^* from using the predicted steady-state value from a standard infinite horizon framework, that is,[4]

$$r^* = \rho + \frac{g_A}{\sigma} + n. \tag{9}$$

This value of r^* can be interpreted as a lower bound for the BK economy. The reason is that under an identical discount rate, a non-altruistic finite-lived agent will accumulate less capital than an infinitely-lived individual because the former's time horizon is shorter. r^* can also be interpreted

[4]This expression is derived later on, in section 4.1.

Table 2: Predicted values of s^* by the BK model, years

	Benchmark	$(\psi, \theta) = (0.28, 0.176)$	$\xi = 0.2$
$r^* = 5.6\%$	15.5	21.5	17.1
$r^* = 7.6\%$	10.9	14.3	12.5
$r^* = 9.6\%$	7.3	7.1	8.6

as the value to which the interest rate converges as agents' life expectancy goes to infinity.

We further assume that $n = 0.016$, the average labor force growth rate in the U.S. during the period 1950-1980. For $\sigma = 1$, and $\rho = \{0.02, 0.04, 0.06\}$ equation (9) generates $r^* = \{5.6, 7.6, 9.6\}$, respectively. Column 2 in Table 2 presents the benchmark average years of schooling predicted under the three alternative values of r^*. Columns 3 and 4 present predicted schooling. under two alternative cases, $(\psi, \theta) = (0.28, 0.176)$ and $\xi = 0.2$, considered in BK.[5]

Results in Table 2 (column 3) suggest that the alternative parameterization of the schooling function ($(\psi, \theta) = (0.28, 0.176)$) implies higher schooling values under $r^* = \{5.6, 7.6\}$. This occurs because lower values of ψ cause the marginal effect of the educational attainment on human capital to decline at a slower rate. That, in turn, leads agents to accumulate more years of schooling. Results in column 4 suggest that when schooling has a positive effect on utility ($\xi > 0$), then predicted schooling values are always higher than the benchmark, as expected.

In order to reflect on the goodness of the above predictions, we need to establish a range of steady-state educational attainment values that is consistent with evidence. Barro and Lee (2000), in an updated version of their 1993 paper, present data on average educational attainment for 142 countries. They report that in year 2000, average educational attainment of the eighteen wealthiest countries ranged from 7.2 years (Italy) to 12.1 years (U.S.). In an earlier study, Psacharopoulos (1994) reported a maximum value of average educational attainment equal to 13.6 for the U.S. Given this evidence, we suggest that a reasonable steady-state educational attainment value may be between 8 and 14 years.[6] Table 2, therefore, implies that the BK model delivers reasonable

[5]To compute $c(s)^{1/\sigma}/[w(s)h(s)]$ in expression (8), we follow the method proposed by BK. That is for $\sigma = 1$, we set this ratio to the annuity value of expected labor-income growth, $1 + r\sum_{t=s}^{T}[(1 + g_A)^{(t-s)} - 1]/(1 + r)^{(t-s)}$.

[6]Since the early 1950s, average educational attainment has been increasing in all industrial countries. For example, Barro and Lee (2000) report that in the U.S. average educational attainment in 1960 was 8.5 years, increasing to 11.9 in 1980, and reaching 12.1 years in 2000. Even though we assume 14 years as a sensible upper bound of educational attainment, higher values that are economically feasible do not change our results qualitatively.

Table 3: Predicted s by the BK model for different values of T, years

		T			
r^*	(ψ, θ)	54.5	100	200	500
5.6%	$(0.58, 0.323)$	15.5	30.0	57.0	84.7
7.6%	$(0.58, 0.323)$	10.9	15.3	17.2	17.3
9.6%	$(0.58, 0.323)$	7.3	8.3	8.5	8.5
7.6%	$(0.28, 0.176)$	14.3	28.7	40.1	42.0
9.6%	$(0.28, 0.176)$	7.1	9.2	9.5	9.5

predictions as long as the interest rate is sufficiently high.

2.2 Increasing the time horizon in the BK model

Given that the infinite horizon model is the main workhorse of modern macroeconomics, we next, we study predictions for s^* in the BK setup as the time horizon (T) increases. It is important to notice that the marginal derivative of the experience function with respect to the worker's age diminishes with this argument and eventually becomes negative. This means that for sufficiently large values of $a - s$, the term $g(a - s)$ becomes negative. In particular, using parameter values in Table 1 obtains that $a - s > 36$ implies that $g\prime(a - s) < 0$, and that $a - s > 72$ implies that $g(a - s) < 0$. This implication of the BK model is not economically sensible when the life-span is relatively high.

In what follows, we redefine the function such that the relevant derivative is never negative. Let $\bar{a}(s)$ be such that $g\prime(\bar{a}) = 0$. Then $g(a - s) = \gamma_1(a - s) + \gamma_2(a - s)^2$ if $a < \bar{a}$; otherwise, $g(a - s) = \gamma_1(\bar{a} - s) + \gamma_2(\bar{a} - s)^2$. The assumption implies that, beyond some age, additional work experience does not generate more human capital, but does not diminish it either. Compared to the original quadratic form, given by expression (7), the suggested modified specification generates lower levels of schooling because experience has a larger return for some values of $(a - s)$.

Table 3 presents predictions of the BK model for values of $T = \{54.5, 100, 200, 500\}$.[7] We see that, as T rises, the predicted steady-state level of schooling years becomes extremely sensitive to changes in the parameter values. In addition, the table shows that, for low levels of the interest rate, predictions are implausibly high when $T = 500$. For example, when the steady-state interest

[7]For $T > 500$, agents' decision about early schooling hardly changes, because the present discounted value of benefits obtained far away in the future are negligible.

rate is 7.6%, which corresponds to a discount rate of 0.04, the steady-state value of s is equal to 17.3 when $\psi = 0.58$, and 42.0 when $\psi = 0.28$. In contrast, for sufficiently large values of r^* future benefits are heavily discounted and agents decide on a small investment in education, regardless of T.

In sum, we have found that the BK setup generates predictions on the number of years of schooling of the workforce that agree with the evidence for some parameterizations. However, as agents' time horizon increases the parameter set that delivers empirically relevant predictions shrinks. We believe that this casts doubts on the appropriateness of the proposed human capital technology when the analysis is carried out in an infinite horizon model. We next explore potential alternatives to the BK specification that are more suitable for infinite horizon models.

3 A Flow Approach to Human capital

In order to reconcile agents' finite schooling levels with the infinite horizon model, papers such as Bils and Klenow (1996) and Jones (1997, 2002), among others, propose interpreting average years of schooling as the fraction of an individual's time endowment allocated to accumulating skill. That is, they interpret average years of schooling as an investment rate (a flow variable).

The economy is identical to the one presented in the previous section except that now each households consist of identical infinitely-lived agents that allocate their time endowment between going to school and working. Newborns are assumed to inherit physical capital that makes all family members possess the same amount of this input. With that assumption, it is no longer needed to keep track of the cohort to which agents belong because all of them make the same decisions, hence it is safe to drop the cohort index a from the model.

The skilled-labor equation that determines the way by which skill is formed and embodied into labor is given by

$$H(t) = e^{f_l[l_h(t)]+g_l[1-l_h(t)]} \left[1 - l_h\left(t\right)\right] L\left(t\right); \tag{10}$$

where L is labor size. Compared to the BK specification given by equations (2) and (3), the schooling $f_l(\cdot)$ and experience $g_l(\cdot)$ components that determine the level of human capital are now functions of l_h. That is, schooling and experience depend on the fraction of each agent's time endowment allocated to education rather than the educational attainment stock s in the BK model. In addition, no external effect from teachers' human capital is allowed. The rest of the time available in every period, $(1 - l_h)$, is supplied as labor.

Consistent with our analysis, we show later on that by reinterpreting l_h as the fraction of an agents' "productive life" spent in school, it becomes a positive function of the average educational attainment level s, i.e. $l_h = l_h(s)$. This establishes a link between f_l and the schooling function f of the BK model, and between g_l and g.

Given neoclassical assumptions, the decentralized and centralized problems obtain the same equilibrium outcomes. For ease of exposition we focus on the central planner's problem.

3.1 Social planner's problem

Let lower case letters denote variables normalized by the size of labor. A central planner would choose the sequence $\{k(t), c(t), l_h(t)\}_{t=0}^{\infty}$ to maximize the lifetime utility of the representative consumer subject to the feasibility constraints of the economy. The problem is stated as follows:

$$\max_{k,c,l_h,l} \int_0^{\infty} e^{-\rho t} \frac{c(t)^{1-1/\sigma} - 1}{1 - 1/\sigma} \, dt$$

subject to,

$$\begin{aligned} y &= k^{1-\alpha} \left[A\, e^{f_l(l_h) + g_l(1-l_h)} (1 - l_h) \right]^{\alpha} \\ \dot{k} &= y - c - (n+\delta)k \qquad (11) \\ \dot{L} &= nL \\ \dot{A} &= A\, g_A \\ & L_0,\ K_0,\ A_0 \ \text{given;} \end{aligned}$$

where δ is the depreciation rate of capital; and L_0, K_0, A_0 are the initial levels of labor, physical capital and technology, respectively.

Equation (11) is the standard law of motion of the stock of per capita physical capital, as well as a feasibility constraint. The FOCs for the interior solution obtain the optimal share of time endowment in education at steady state as

$$l_h^* = 1 - \frac{1}{f_l'(l_h^*) - g_l'(1 - l_h^*)}. \qquad (12)$$

Equation (12) states that at the margin, the cost of investing one more unit of labor in education must equal its benefit, which is the increase in effective labor from additional schooling in the output sector. Notice that the opportunity cost of schooling includes the marginal benefit from

Table 4: Predictions of the flow model

Case	l_h^*	s^*
Benchmark	0.49	26.2
$(\psi, \theta) = (0.280, 176)$	0.60	32.2
$(\gamma_1, \gamma_2) = (0.0934, -0.0011)$	0.41	21.8

experience $g\prime$. The above equation also implies that human capital investment may not occur if the returns to schooling are not sufficiently large.

3.2 Calibration and predictions of the flow model

Next, we examine the model's steady-state predictions regarding the schooling variable, l_h^*. Following Jones (2002), we impose the following explicit function that maps the population's average years of schooling, s, into the investment variable l_h:

$$l_h = \frac{s}{N}. \tag{13}$$

The variable l_h is, interpreted as the fraction of an agents' productive life (call it N) spent in school.[8] The relationship between l_h and s given by equation (13) is certainly true at steady state. Notice that the optimal schooling decision, given by equation (12), can be written as a function of the Mincerian return to education because equations (6) and (13) allow for the following equality:

$$f_l'(l_h) = f'\left[s(l_h)\right] = f'(s)\frac{\partial s}{\partial l_h} = f'(s)N = \theta N^{1-\psi} l_h^{-\psi}. \tag{14}$$

In the same way, for the purpose of calibration, we reinterpret $1 - l_h$ as the fraction of the individuals' productive life in the work-place accumulating experience. Then, if we equalize a in function (7) to N, the return to life-time experience can be written as

$$g_l'\left(1 - l_h\right) = g'(N - s)\frac{\partial(N - s)}{\partial(1 - l_h)} = g'\left(N - s\right)N = N\left[\gamma_1 + 2\gamma_2\left(1 - l_h\right)N\right]. \tag{15}$$

We can now solve for l_h^* using equations (12), (14) and (15).[9] To obtain numerical predictions, we equalize N to the number of productive-life years considered by BK, 54.5. This figure equals

[8] N is introduced for the purpose of calibration and assigning values to $f_l'(l_h^*)$ and $g_l'(1 - l_h^*)$.

[9] For the calibrated parameter values, the RHS of equality (12) is strictly decreasing in the range of economically feasible values of l_h, that is, in the zero-one interval. As a consequence, the solution to the system of equations turns out to always be unique. Once we have the solution for l_h^*, the value of s^* is recovered using equation (13).

the average life expectancy 60.5 in Barro and Lee's (1993) sample minus 6 years, the typical pre-schooling period. The rest of parameters take on the benchmark values in Table 1. Predictions of average years in schooling are given in Table 4. For this parameterization, agents spent at steady state 49 percent of their time at school, which corresponds to a value of s^* of 26.2 years. This number is certainly much higher than our acceptable range of $8 - 14$ years.

The results are certainly sensitive to the choice of parameters such as θ, ψ, γ_1, γ_2 and N. Table 4 also presents predictions for the second set of schooling function parameters $((\psi, \theta) = (0.28, 0.176))$ considered in BK. As expected, under this case the equilibrium value of s^* increases even more to 44.1 years. On the other hand, we know that the optimal value of l_h^* should decrease with the return to experience – recall equation (12). As a robustness check of our results, we then take the average values of the experience function coefficients for the 5 countries with the largest γ_1. Our calculations obtain $\gamma_1 = 0.0934$ and $\gamma_2 = -0.00110$.[10] Table 4 reveals that the optimal number of years of education in this case is 21.8, once again implausibly high. Another parameter to which the predictions of the model are sensitive is the agents' productive life, N. It is easy to show that the educational attainment s^* declines with N. In the flow model, using the benchmark values, the variable s^* equals 14 (our upper bound) only when $N = 35$. However, this value of N is too small compared to the general consensus.

These findings suggest that the skilled-labor specification under a flow approach is not consistent with evidence. Even in richer models with monopolistic competition and R&D, or including unskilled along with skilled labor, this conclusion does not change. The reason is that the human capital specification without dynamic components always delivers the optimal investment in schooling given by equation (12). Put differently, marginal costs and benefits of education remain the same or vary proportionally in more elaborated models.[11]

[10] A value of γ_1 equal to 0.0934 represents the average in the BK sample plus 1.96 standard deviations. It is also important to notice that, in the BK sample, larger values of γ_1 are generally associated with lower values of γ_2, with a correlation coefficient between the two parameters of -0.61.

[11] For example, in a richer model with monopolistic competition and an R&D sector, we can consider a law of motion for technology as in Perez-Sebastian (2000), $\dot{A} = \mu A^{\eta} H_A^{\lambda} (A^w/A)^{\varphi}$; where $H_A = e^{f_l[l_h(t)] + g_l[1 - l_h(t)]}\, l_A\, L$ is the skilled-labor input employed in R&D; and A^w is the stock of existing technology in the world that grows at an exogenous rate g_{A^w}. In this model, a fraction of labor $1 - l_A - l_h$ is allocated to final output production. Solving the social planner problem obtains equation (12), that is, the R&D-based model's predictions about educational attainment are the same as those of the two-sector neoclassical growth model.

Alternatively, we can extend the model's production technology to relax the assumption that unskilled and skilled labor are perfect substitutes. We can, for example, substitute (1) with the following generalized CES specification (see Arrow *et al.*, 1961, for details): $Y = K^{1-\alpha}\{A[zL_u^{\upsilon} + (1-z)[e^{f_l[l_h(t)] + g_l[1 - l_h(t)]}\,(1 - l_h)\, L_s]^{\upsilon}]\}^{\alpha/\upsilon}$; where L_u is the number of unskilled workers who allocate zero time in schooling; and L_s is the number of skilled workers, who allocate a fraction of their time, l_h, to schooling. The FOC form the social planner problem is once again given by

4 An Alternative Specification of Skilled-Labor

In this section, we offer an alternative skilled-labor specification with dynamic components that delivers educational attainment levels that are consistent with those in the data. An attractive feature of the proposed specification is that it does not include additional variables such as work experience or teachers' positive effect on utility, which makes it easy to incorporate into existing growth models.

The economy is populated with an exogenously growing (n) number of infinitely-lived agents. Final output production is once again given by equation (1). We consider the most simple version of the BK human-capital specification. The form that we use in our alternative model also corresponds to the one incorporated into the aggregate production function by recent papers such as Hall and Jones (1999). In particular, the economy-wide stock of human capital per capita is now given by

$$h\,(t) = e^{f[\bar{s}(t)]}; \tag{16}$$

where $\bar{s}$ is workers' average educational attainment. Notice that, in (16), human capital is affected by the average efficiency level, instead of by the sum across individuals of efficiency levels considered in specification (3). The new formulation then introduces an externality. Labor productivity depends on the average educational attainment in the economy, and a worker's investment in education, which has positive external effects on other workers' productivity. This positive externality is consistent with evidence surveyed, for example, by Becker (1975). It works out that its inclusion also simplifies algebra in the proposed model.

At period t, agent i invests a fraction $l_h(t,i)$ in schooling. The average educational attainment in the economy is then given by

$$\bar{s}(t) = \frac{1}{L(t)} \int_0^t \left(\int_0^{L(j)} l_h(j,i)\; di \right)\; dj. \tag{17}$$

This expression is the sum of the time spent in school by all agents since the beginning period zero, divided by the size of the population at date t, $L(t)$. If we interpret each period as one year then $\bar{s}(t)$ corresponds to the average years of schooling of the workforce at date t in an economy where people are infinitely lived.

Differentiating expression (17) with respect to t delivers the following law of motion of $\bar{s}$:

$$\bar{s}(t) = l_h^e(t) - n\,\bar{s}(t); \tag{18}$$

expression (12).

where $l_h^e(t) = [1/L(t)]\,[\int_0^{L(t)} l_h(t,i)\; di]$ is the economy-wide fraction of time allocated to acquiring skills. This law of motion is equivalent to the standard motion of physical capital. Equation (18) states that, in the aggregate, workers' average educational attainment increases with the fraction of time that agents invest in schooling, but (exogenously) declines with the increase in population.

4.1 Social planner's problem

Even though the new specification incorporates an externality, for simplicity we keep focus on the centralized solution. The social planner problem is as follows:

$$\max_{\{k,c,l_h,S\}} \int_0^\infty e^{-\rho t}\, \frac{c(t)^{1-1/\sigma} - 1}{1 - 1/\sigma}\, dt$$

subject to,

$$y = k^{1-\alpha} \left[A\, e^{f(\bar{s})}\, (1 - l_h^e) \right]^{\alpha}$$

$$\dot{k} = y - c - (n + \delta)\, k$$

$$\bar{s} = l_h^e - n\, \bar{s}$$

$$\dot{L} = n\, L$$

$$\dot{A} = A\, g_A$$

$$L_0,\ K_0,\ A_0,\ \bar{s}_0 \ \text{ given.}$$

We construct the Hamiltonian, and get the FOCs for the interior solution. After some algebra we show that the Euler equation that characterizes the optimal allocation of labor to human capital investment is given by

$$f'(\bar{s})\,(1 - l_h^e) + \left[\frac{\dot{y}}{y} + \frac{\dot{l}_h^e}{1 - l_h^e} \right] = r; \tag{19}$$

where,

$$r = (1-\alpha)\frac{y}{k} - \delta = \rho + \frac{\dot{c}/c}{\sigma} + n. \tag{20}$$

Equation (20) is the standard Euler condition for consumption that at steady state reduces to (9). Expression (19) can be interpreted as an arbitrage condition. The LHS of equation (19) is the return to sacrificing one unit of output for acquiring schooling. The first term represents the dividend from the increase in effective labor. The term in brackets represents the capital gain/loss, which is equal to the percentage change in the price of education – notice that the shadow price of having additional units of education equals the marginal productivity of labor in output production,

Table 5: Predictions in alternative model

		$\sigma = 1$		$\sigma = 1/2$	
ρ	(ψ, θ)	l_h^{e*}	$\bar{s}^*$	l_h^{e*}	$\bar{s}^*$
0.02	$(0.58, 0.323)$	0.34	21.2	0.21	13.3
0.04	$(0.58, 0.323)$	0.21	13.3	0.15	9.1
0.06	$(0.58, 0.323)$	0.15	9.1	0.11	6.6
0.04	$(0.28, 0.176)$	0.30	18.4	0.17	10.9
0.06	$(0.28, 0.176)$	0.18	10.9	0.10	6.4

$\frac{\alpha y}{1-l_h}$. The RHS of equation (19) captures the opportunity cost of schooling investment, given by the interest rate. In equilibrium, both sides must be equalized.

Let g_j be the steady-state growth rate of variable j. Along the balanced growth path $g_y = g_c = g_A$, and $\dot{l}_h^e = 0$. Euler equations (19) and (20) then imply that the optimal share of labor in schooling is given by

$$l_h^{e*} = 1 - \left[\frac{n + \rho + (1/\sigma - 1)\ g_A}{f'(\bar{s}^*)}\right]. \tag{21}$$

The optimal education investment depends directly on its current return $f'(\bar{s}^*)$ and future benefits of applying the acquired knowledge (g_A). As expected, the optimal education investment declines with the preference parameters ρ and $1/\sigma$, and the population growth rate n.

Since at steady state l_h^e remains constant, the motion equation (18) implies that so does $\bar{s}^*$ – otherwise $g_{\bar{s}}$ can not be a constant – and therefore

$$\bar{s}^* = \frac{l_h^{e*}}{n}. \tag{22}$$

Using expression (6), equation (21) can now be written as

$$l_h^{e*} = 1 - \left[\frac{n + \rho + (1/\sigma - 1)\ g_A}{\theta\left(\frac{l_h^{e*}}{n}\right)^{-\psi}}\right]. \tag{23}$$

4.2 Calibration and predictions of the alternative model

We use again the BK benchmark values of the parameters presented in Table 1, and 0.016 for n. We choose $\sigma \in \{1/2, 1\}$ from the existing literature.[12] We then use equation (23) to generate values

[12]These estimates of the intertemporal elasticity of substitution, σ, are taken from Hall (1988), Attanasio and Weber (1993), and Guvenen (2002).

for l_h^{e*}, and expression (22) to recover $\bar{s}^*$. It is easy to show that equation (23) has a unique root but does not have an analytic solution when $0 < \psi < 1$. We therefore resort to numerically solving for l_h^{e*}.

Table 5 presents the estimates of l_h^{e*} and $\bar{s}^*$ resulting from this calibration exercise. Consistent with our previous calibrations, we consider three different values for the discount rate $\rho = \{0.02, 0.04, 0.06\}$ that correspond to interest rates $r^* =\{5.6, 7.6, 9.6\}$, respectively, when $\sigma = 1$. Hence, the forth column in Table 5 can be compared directly with the last column of Table 3. We see that predictions in the infinite horizon model with our proposed human capital specification are sensitive to changes in the parameter values, but substantially less so than in the BK model. In addition, for $\sigma = 1$, predictions on $\bar{s}^*$ are all plausible except when the discount rate is relatively low, i.e. $\rho = 0.02$. When $\sigma = 1/2$, on the other hand, we have the opposite result: predictions on $\bar{s}^*$are all plausible except when ρ are large, i.e. $\rho = 0.06$.[13]

The main finding here is that our calibrated model is able to deliver levels of educational attainment consistent to those in the data (i.e. $\bar{s}^* \in [8, 14]$). Compared to the flow specification, this is the result of the dynamic structure of the proposed human capital accumulation technology that is able to reduce sufficiently the overall returns to education. Plausible predictions are not obtained for all sensible parameterizations of the model, but results indicate that this problem is less severe than with the BK specification. As a consequence, we argue that our proposed human capital technology represents an improvement over the BK technology when used in the infinite horizon model.

5 Conclusion

This paper has explored different specifications of the human capital technology that follow the Mincerian approach proposed by the seminal contribution of BK. In particular, we have searched for specifications that can match existing data on educational attainment with economic growth models. We have first examined the robustness of the specification adopted by BK in their finite-horizon setup. We have shown that predictions about workers's average years of schooling become extremely sensitive to changes in the parameter values when the life expectancy of individuals is infinite. In addition, BK delivers implausibly large values of the educational attainment if interest

[13] A sensitivity analysis shows that our results are robust to changes in other parameters such as g_A and n.

rates are not sufficiently large.

We have then searched for specifications that could be more appropriate in the infinite-horizon framework. The first attempt considered a human capital specification proposed in recent papers in a simple two-sector growth model. Calibrating the model reveals several caveats associated with this specification that follows a flow approach, and casts doubts in its use in theoretical and empirical work. In particular, the model predicts that the optimally chosen educational attainment levels are significantly higher than the those observed in the data.

Finally, we have presented an alternative specification of human capital within a standard infinite-horizon neoclassical growth model that incorporates an explicit law of motion of the mean years of education. An important feature of the proposed specification is that it does not include additional variables such as work experience, which makes it easy to incorporate into existing theoretical growth models and easy to adopt in growth accounting exercises. Simple calibration exercises of the model at steady state with parameters found in the literature reveal that the proposed specification of human capital is more successful in replicating the observed data on educational attainment.

A parameter to which the predictions of the *flow-approach* model are especially sensitive, is agents' productive life, N. Our sensitivity analysis on N reveals an important weakness of this approach to human capital specification. The model takes the market return to human capital investment as contemporaneous and immediate, whereas in practice the return to schooling is accrued over agents' lifetime. This leads to overstating the return to schooling, and predicting unrealistically high educational attainment levels. A substantial reduction in agents' productive life is a necessary adjustment to reduce the return to schooling in the model.

In contrast, our proposed human capital specification does not require experience, or lower productive life to obtain desirable predictions. This is because when human capital accumulates, preference parameters such as ρ and σ that determine intertemporal decisions, discount the return to schooling sufficiently to deliver sensible values of the average educational attainment. Under the infinite horizon framework, our proposed human capital specification is still sensitive to alternative parameterizations but significantly less so than BK's specification. Another attractive feature of the proposed human capital specification is that it may be used for off-steady-state analysis. Further work with it revealed that outside the balanced growth path, enrollment rates and the average educational attainment are negatively correlated, which is the relationship that dominates the data (see e.g., Pritchett (1997) pp. 27-30).

There are several conclusions of the paper for future research. Incorporating the proposed human capital specification into an R&D-based model and examining the transitional dynamic properties of the model is a promising next step. Using the reduced form equation implied by our alternative specification, one could also reexamine accounting exercises such as those in Klenow and Rodríguez-Clare (1997a), Hall and Jones (1999) and Temple (2001). In addition, we could extend our simplistic model to consider other potentially important factors of human capital dynamics that have been kept outside the present analysis. For example, other means of human capital accumulation such as on-the-job training or the production of education at home have been emphasized by the literature. Allowing for these additional engines of human capital accumulation would likely result in lower formal educational attainment. Variables that affect the return to schooling would also have an impact on the model predictions. Human capital depreciation is one of these variables, and its incorporation would also tend to reduce the predicted educational attainment.

A more generalized production function that permits imperfect labor substitutability could also affect the results provided by our human capital technology. In this case, however, the impact of considering unskilled and skilled labor as imperfect substitutes is less clear. Finally, we have focused on the socially optimal equilibrium. Predictions with the proposed specification would differ under the decentralized problem. In particular, agents would not internalize the positive external effects and therefore choose less schooling. All these aspects deserve further research that we leave for future research.

References

Arrow, K., Chenery, H., Minhas, B. and Solow, R., "Capital–Labor Substitution and Economic Efficiency," *Review of Economics and Statistics*, 43:225–250, 1961.

Attanasio, O. and Weber, G., "Consumption, the Interest Rate and Aggregation," *Review of Economic Studies*, 60:631–649, 1993.

Barro, R. and Lee, J.,"International Comparisons of Educational Attainment," *Journal of Monetary Economics*, 32:363–394, 1993.

– – and – –, "International Data on Educational Attainment: Updates and Implications," *CID working paper#42*, 2000.

Becker, G.S., *Human Capital: Second Edition*, Columbia University Press, 1975.

Bils, M. and Klenow, P., "Does Schooling Cause Growth or the Other Way Around?" *working paper*, University of Rochester, 1996.

– – and – –, "Does Schooling Cause Growth?" *American Economic Review*, 90:1160–1183, 2000.

Guvenen, M.F., "Reconciling Conflicting Evidence on The Elasticity of Intertemporal Substitution: A Macroeconomic Perspective" *working paper*, Carnegie Mellon University, 2002.

Hall, R., " Intertemporal Substitution in Consumption," *Journal of Political Economy*, 96:339–357, 1988.

– – and Jones, C., "Fundamental Determinants of Output Per Worker Across Countries," *Quarterly Journal of Economics*, 114:83–116, 1999.

Heckman, J. and Klenow, P., "Human Capital Policy," *working paper*, University of Chicago, 1997.

Jones, C.I., "Convergence Revisited," *Journal of Economic Growth*, 2:131–153, 1997.

– –, "Sources of U.S. Economic Growth in a World of Ideas," *American Economic Review*, 92(1):220-239, March 2002.

– –, "Human Capital, Ideas, and Economic Growth," in Paganetto, L. and E.S. Phelps (ed.), *Finance, Research, Education, and Growth* (New York: Palgrave), 2003.

Klenow, P. and Rodríguez–Clare, A., "Economic Growth: A Review Essay," *Journal of Monetary Economics*, 40:597–617, 1997a.

Klenow, P. and Rodríguez–Clare, A., "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" *NBER Macroeconomics Annual*, (eds) B. Bernanke and J. Rotemberg ed., MIT Press, 73:102, 1997b.

Krueger, A. and Lindahl, M., "Education for Growth: Why and for Whom?," *Journal of Economic Literature*, 39:1101-1136, 2001.

Mulligan C.B., and Sala-i-Martin, X., "Measuring Aggregate Human Capital," *Journal of Economic Growth*, 5:215-252, 2000.

Perez-Sebastian, F., "Transitional Dynamics in an R&D–Based Growth Model with Imitation: Comparing its Predictions to the Data," *Journal of Monetary Economics*, 45:37–61, 2000.

Pritchett, L., "Where Has All the Education Gone?" *World Bank working paper#1581*, 1997.

Psacharopoulos, G., "Returns to Investment in Education: A Global Update," *World Development*, 22:1325–1343, 1994.

Schultz, T.P., *The Economic Value of Education*, Columbia University Press, 1963.

– –, "Returns to Women Education," Population, Health, and Nutrition Department, *World Bank working Paper#89/001*, 1989.

Solow, R.M., "Neoclassical Growth Theory," Handbook of Macroeconomics, (eds) J.B. Taylor and M. Woodford,

Stokey, N.L., "Learning by Doing and the Introduction of New Goods," *Journal of Political Economy*, 96:701–717, 1988.

Temple, J.R.W., "Generalizations That Aren't? Evidence on Education and Growth," *European Economic Review*, 45:905–918, 2001.

Wößmann, L., "Specifying Human Capital," *Journal of Economic Surveys* 17(3):239-270, 2003.