Online Supplementary Material for
Diseases, infection dynamics and development

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A Simpler Model

This *Online Supplementary Material* presents a simpler version of the model that appears in the paper. There are three key differences: prevention and disease transmission are simpler, preferences are linear and the production technology assumes both capital and labor are necessary inputs.

Disease Environment

Young individuals can undertake one of two types of preventive health investment, $x_t \in \{0, x\}$. Diseases spread from infected older individuals to susceptible younger ones. A susceptible young person randomly meets $\mu > 1$ older individuals during the first half of his youth, before old infected agents start dying. Not all of these older individuals will be infected and not all encounters with infected people result in transmission. For the time being, let us assume that the probability of being infected (p_t) after these μ encounters is given by

$$p(x_t) = \mu\pi(x_t)i_t, \tag{1}$$

where $\pi(x_t)$ is the probability that a young individual gets infected in an encounter with an infected adult, and i_t is disease prevalence among adults. Furthermore, suppose that

$$\pi(x_t) = \begin{cases} \pi_1, & \text{if } x_t = x > 0 \\ \pi_0, & \text{if } x_t = 0. \end{cases} \tag{2}$$

Let $\mu\pi_1 < 1 < \mu\pi_0$, that is, an infected person infects more than one susceptible person in the absence of prevention but less than one (on average) if susceptible populations engage in prevention.

Equation (1) exhibits a negative externality that characterizes communicable diseases. When an individual chooses preventive health investment *ex ante* – before he meets an infected person – he does not take into account how his decision impacts the susceptibility of future generations. Furthermore, this externality is amplified by the random matching process.

Technology

A continuum of firms operate in perfectly competitive markets to produce the final good using capital (K) and efficiency units of labor (L). The firm-specific technology is CRS in private input but exhibits learning-by-doing externalities

$$F(K^i, L^i) = A(K^i)^\alpha(\bar{k}L^i)^{1-\alpha}, \tag{3}$$

where A is a constant productivity parameter, and $\bar{k}$ denotes the average capital per effective unit of labor across firms. To generate a balanced-growth path with strictly positive growth we assume that $(1-\alpha)A > 1$.

Standard factor pricing relationships under such externalities imply that the wage per effective unit of labor (w_t) and interest factor (R_t) are given respectively by

$$w_t = (1-\alpha)Ak_t \equiv w(k_t), \tag{4}$$

$$R_t = \alpha A \equiv R. \tag{5}$$

Preferences and Decisions

Suppose each individual is risk neutral. An uninfected individual maximizes lifetime utility

$$c^U_{1t} + \beta c^U_{2t+1}, \ \beta \in (0,1), \tag{6}$$

subject to the budget constraints

$$c^U_{1t} = w_t - x_t - z^U_t \tag{7}$$

$$c^U_{2t+1} = R_{t+1} z^U_t, \tag{8}$$

where z denotes savings and x is given by decisions made early in period t.

An infected individual faces a constant probability $\phi \in (0,1)$ of surviving from the disease before reaching old-age. Assuming zero utility from death, he maximizes expected lifetime utility

$$\delta\left[c^I_{1t} + \beta\phi c^I_{2t+1}\right], \tag{9}$$

subject to

$$c^I_{1t} = (1-\theta)w_t - x_t - z^I_t \tag{10}$$

$$c^I_{2t+1} = R_{t+1} z^I_t + \tau_{t+1}. \tag{11}$$

Suppose $\beta R > 1 > \phi\beta R$. Under this condition, infected individuals do not save at all while uninfected individuals save their entire labor income. That is, $c^U_{1t} = c^I_{2t+1} = z^I_t = \tau_{t+1} = 0$, $c^I_{1t} = (1-\theta)w_t - x_t$, $z^U_t = w_t - x_t$, and $c^U_{2t+1} = R_{t+1}(w_t - x_t)$. Substituting these into expected lifetime utility gives the two indirect utility functions

$$V^U(x_t) = \beta R_{t+1}(w_t - x_t), \text{ and} \tag{12}$$

$$V^I(x_t) = \delta\left[(1-\theta)w_t - x_t\right]. \tag{13}$$

At the beginning of t, adults choose the optimal level of x_t to maximize expected lifetime utility. Recall that a young individual's probability of catching the disease is p_t given by (1). Hence, individuals choose x_t to maximize

$$p_t(x_t)V^I(x_t) + [1 - p_t(x_t)]\, V^U(x_t), \tag{14}$$

at the beginning of period t. Given that prevention is either investing x or nothing at all, the optimal decision is x if and only if expected lifetime utility is higher in doing so

$$p_t(x)V^I(x) + [1 - p_t(x)]\, V^U(x) > p_t(0)V^I(0) + [1 - p_t(0)]\, V^U(0). \tag{15}$$

All savings are invested in capital which are rented out to final goods producing firms, earning the rental rate. The initial old generation is endowed with a stock of capital K_0 at $t = 0$. The depreciation rate on capital is set equal to one. Finally, an exogenously specified fraction i_0 of old agents are infected.

Dynamics

The dynamic behavior, as in the submitted version, is fully described by the pair of difference equations

$$k_{t+1} = \frac{[1 - p(k_t, i_t)]z^U(k_t, i_t)}{1 - \theta p\left(p(k_t, i_t)\right)}. \tag{16}$$

and

$$i_{t+1} = p(k_t, i_t). \tag{17}$$

given initial conditions $\{k_0, i_0\}$.

The phase diagram appears in Figure 1. Three loci in (k_t, i_t) space determine the dynamics. The first two consist of the locus along which disease prevalence remains constant ($\Delta i_t = 0$) and the locus for which capital per effective unit of labor remains constant ($\Delta k_t = 0$). The third locus is the general equilibrium version of (15) which separates positive preventive investment from zero investment.

The assumption $\mu\pi_1 < 1 < \mu\pi_0$ implies, from equations (1) and (17), that for no (k_t, i_t) with $i_t \in (0, 1)$ is disease prevalence constant. Specifically, disease prevalence increases when $x_t = 0$ and decreases if $x_t = x$. From expressions (1), (5), (12) and (13) we can rewrite (15) as

$$\mu\pi_1 i_t \delta\left[(1 - \theta)w_t - x\right] + (1 - \mu\pi_1 i_t)\, \beta R\,(w_t - x) > \mu\pi_0 i_t \delta(1 - \theta)w_t + (1 - \mu\pi_0 i_t)\, \beta R w_t.$$

Substituting for equilibrium wages from (4) we obtain

$$i_t > \frac{\beta R x}{(1 - \alpha)Ak_t\mu\left(\pi_0 - \pi_1\right)\left[\beta R - \delta(1 - \theta)\right] + \mu\pi_1\left(\beta R - \delta\right)x} \equiv \Phi(k_t). \tag{18}$$

It is easy to show that Φ is decreasing in k_t, $\lim_{k\to\infty} \Phi(k) = 0$, and $\Phi(0) = \beta R/[\mu\pi_1\left(\beta R - \delta\right)] > 1$. Hence the locus $i_t = \Phi(k_t)$ is downward sloping with a positive intercept exceeding 1 at $k = 0$. The prevalence rate always rises for (k_t, i_t) pairs below and to the left of this locus and always decreases to the right of it.

We next determine the shape of the $\Delta k_t = 0$ locus depending on whether or not individuals invest in preventive care.

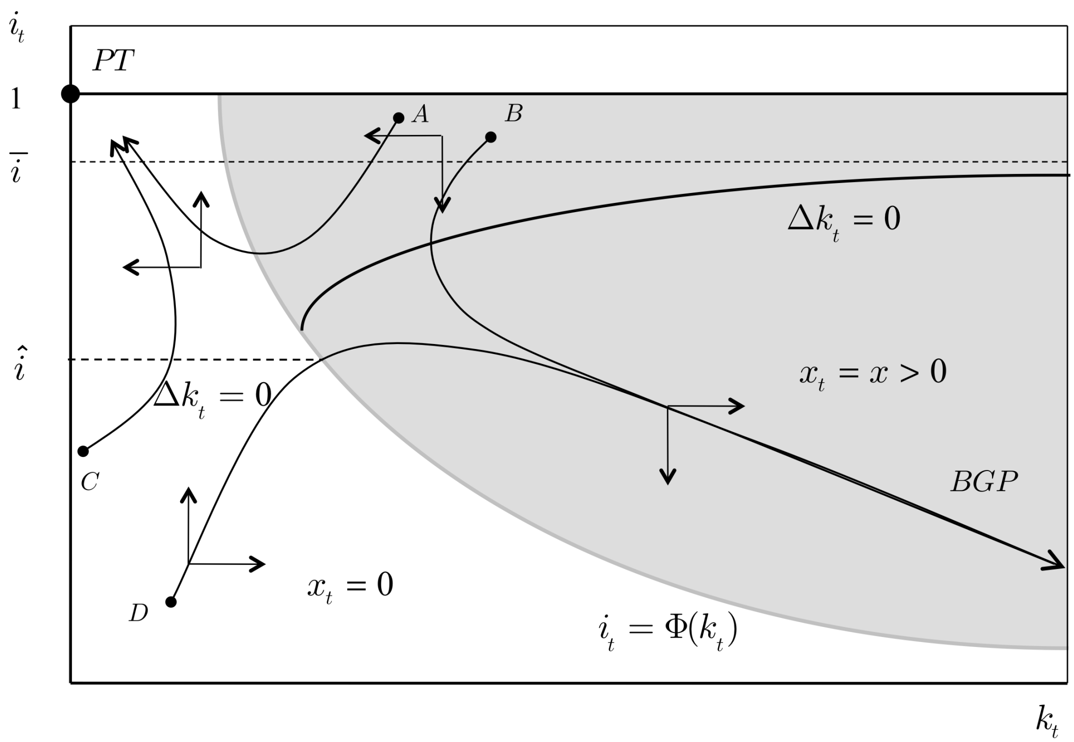

Figure 1: Phase Diagram for the Simpler Model

Case 1: $x_t = 0$

Equations (1) – (4) and (16) imply

$$k_{t+1} = \left[\frac{1 - \mu\pi_0 i_t}{1 - \theta\mu\pi_{t+1}(\mu\pi_0 i_t)} \right] (1-\alpha)Ak_t,$$

so that

$$k_{t+1} \geq k_t \Longleftrightarrow \frac{(1-\alpha)A - 1}{\left[(1-\alpha)A - \theta\mu\pi_{t+1}\right]\mu\pi_0 i_t} \geq 1.$$

Let $(1-\alpha)A > \theta\mu\pi_0$ which ensures that the expression on the left is positive. Note that $\mu\pi_0 i_t$ goes to $\mu\pi_0 > 1$ as $i_t \to 1$. Hence the expression on the left becomes smaller than 1 for a sufficiently large disease prevalence. In addition, the expression becomes undefined as $i_t \to 0$. Hence there exists $\hat{\imath} \in (0,1)$ such that $\Delta k_t < 0$ for all $i_t > \hat{\imath}$ and $\Delta k_t > 0$ for all $i_t < \hat{\imath}$. This threshold is given by

$$\hat{\imath}(\pi_{t+1}) = \frac{(1-\alpha)A - 1}{\left[(1-\alpha)A - \theta\mu\pi_{t+1}\right]\mu\pi_0}. \tag{19}$$

The expression on the right in (19) depends on π_{t+1} and hence on x_{t+1}. As long as the economy is not too close to the $i_t = \Phi(k_t)$ line, time-t dynamics will place the economy to the left of this curve at $t+1$. In this case $\pi_{t+1} = \pi_0$. If, on the other hand, the economy is close to the $i_t = \Phi(k_t)$ line, i_{t+1} might fall to the right of this locus and $\pi_{t+1} = \pi_1$. The threshold $\hat{\imath}$ can therefore be discontinuous with $\hat{\imath}(\pi_1) < \hat{\imath}(\pi_0)$.

Case 2: $x_t = x$

Now the motion equation of capital (16) becomes

$$k_{t+1} = \left[\frac{1 - \mu\pi_1 i_t}{1 - \theta\mu\pi_{t+1}(\mu\pi_1 i_t)} \right] \left[(1-\alpha)Ak_t - x\right],$$

which implies

$$k_{t+1} \geq k_t \Longleftrightarrow k_t \geq \frac{(1-\mu\pi_1 i_t)x}{(1-\mu\pi_1 i_t)(1-\alpha)A - \left[1 - \theta\mu\pi_{t+1}(\mu\pi_1 i_t)\right]} \equiv \Psi(i_t;\pi_{t+1}). \tag{20}$$

The denominator of Ψ is positive at $i_t = 0$ and declines monotonically with i_t. Hence Ψ increases with i_t. But the denominator of Ψ can become zero if

$$i_t = \bar{\imath} = \frac{(1-\alpha)A - 1}{\left[(1-\alpha)A - \theta\mu\pi_1\right]\mu\pi_1}. \tag{21}$$

We obtain $\bar{\imath}$ by setting π_{t+1} equal to π_1 in the denominator of Ψ. This is the relevant probability since as the denominator approaches zero, Ψ goes to infinity and so does k_t on the $\Delta k_t = 0$ line and hence, preventive investment remains positive for any prevalence rate. Clearly $\bar{\imath} < 1$ if and only if $(1-\alpha)A < 1 + \mu\pi_1(1-\theta\mu\pi_1)/(1-\mu\pi_1)$. Under this assumption, the $\Delta k_t = 0$ line is upward sloping with an asymptote at $\bar{\imath}$.

If $\bar{\imath} > 1$, on the other hand, k_t rises with i_t along the $\Delta k_t = 0$ line and coincides with the $i_t = 1$ line for capital stocks exceeding $(1-\mu\pi_1)x/[(1-\mu\pi_1)(1-\alpha)A - 1 + \theta\mu^2\pi_1^2]$. As before a discontinuity is possible near the $i_t = \Phi(k_t)$ locus. In particular, if i_{t+1} falls to the left of this locus for any (k_t, i_t) on the $\Delta k_t = 0$ schedule, then $\pi_{t+1} = \pi_0$ and k_t will be significantly smaller than if $\pi_{t+1} = \pi_1$ by (20).

The phase diagram ignores these discontinuities in the $\Delta k_t = 0$ schedule since they have minor effects on long-run dynamics. Two stable attractors are present in Figure 1. The first is a zero-growth poverty trap (PT), and the second is a balanced growth path (BGP) with strictly positive growth. There is no preventive investment in the poverty trap and disease prevalence is widespread. In addition, the capital stock in this trap is zero since infected individuals do not save. Economies that move along BGP, on the other hand, invest every period in prevention and asymptotically approach full eradication of infectious diseases and, from equation (20), a growth rate of output per worker equal to $(1-\alpha)A-1$.

Recall that at $t = 0$ the economy is endowed with K_0 units of capital owned by the initial old generation as well as with i_0, the prevalence rate of that generation. Hence both k_0 and i_0 are predetermined variables. Which of the stationary equilibria our model economy gravitates towards partly depends on these initial conditions. Economies that converge to PT are like A and C in Figure 1 with relatively low capital stock or large prevalence rates initially. Economies such as B and D start with more favorable initial conditions to converge to BGP.

One aspect of the dynamics worth emphasizing is that PT is a standard trap in that financial aid can propel an economy from it to the balanced growth path. For example, if an economy located at PT receives a grant that pushes it to a capital stock comparable to B, the economy will start converging to the BGP attractor. This also implies that a relatively capital rich nation can never fall into PT. These characteristics of the poverty trap does not generalize to the model presented in the paper.